Inverse Pignistic Probability Transforms

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Abstract – *In some information fusion processes, the incomplete information set can be naturally mapped into a belief theory information set and a Bayesian probability theory information set. For decision making, the mapping of the belief theory fusion results represented by the basic belief assignment to a probability set is accomplished via a pignistic probability transform. This article introduces the inverse pignistic probability transforms (IPPT) that map the posteriori probabilities into the belief function theories, basic belief assignments. Also introduced are two infinite classes and some finite classes of mapping the posteriori probability results to the basic belief assignment of the belief theory.*

Keywords: *Information Fusion, Estimation, Basic belief assignments, Pignistic probability*

1 Introduction

For information fusion systems where the incomplete information set is naturally mapped into a belief theory information set and a probability theory information set, the need arises to map the belief theory results into probability theory results and vice versa. Smets [1] demonstrated a process of computing a pignistic probability from belief theory results represented by the basic belief assignments (BBAs). Sudano [2] introduced four new pignistic probability transforms that refine the probability estimates. This article introduces the inverse pignistic probability transforms that map the posteriori probabilities into the belief function theories, basic belief assignments. Also introduced are some classes of mapping the posteriori probability results to the basic belief assignment of the belief theory.

2 Background [2]

Let Ω be the set of possible outcomes, where the outcomes are mutually exclusive and exhaustive singleton elements of the decision environment. In Bayesian formalism, the probabilities are assigned only to the singleton subsets of the quantitative information set. These probabilities are used to make the decisions.

For systems with a complex input (e.g., real-time sensor measurements, multidimensional filtered feature extractions, real-time data base and *a priori* data base information content, natural language text and symbols parsing evidence, quantitative and qualitative communication clues, and inconsistent errors), a $\text{Power-set}(\Omega) = 2^{\Omega}$ representation of the outcomes and a two-level (lower & upper) probability portrayal is a better representation of the incomplete information set; i.e., some sensor measurements of attributes populate more than one hypothesis.

Belief theories maintain a two-level probabilistic portrayal of the information set: the Belief or credal level and the Plausibility level. The primary foundation in any decision proposition A_i is the value of the Belief Bel(A_i), while the plausibility $Pl(A_i)$ provides the secondary support for the decision.

The basic belief assignment $\mathrm{m}(\mathrm{A}_J)$ represents the strength of all the incomplete information set for the outcome A_J. The assignments of the BBAs values to a specific subset $A_J \in 2^{\Omega}$ are constrained by the normalization constraint equation.

$$\sum_{A_J \in \ 2^{\Omega}} m(A_J) = 1$$

Using the BBAs as the representative of the incomplete information set, the Belief function can be computed. The Belief of the subset preposition A_J is the sum $m(A_K)$ for all the subsets of A_K contained in A_J .

$$Bel(A_J) = \sum_{A_K \subseteq A_J} m(A_K)$$

The Plausibility of the subset A_J is the sum $m(A_K)$ for all subsets A_K that have a non-null intersection of A_J .

$$Pl(A_J) = \sum_{A_K \cap A_J \neq 0} m(A_K)$$

For any singleton proposition A_i, the probability is bound between the Belief and the Plausibility.

$$\mathrm{Bel}(\mathrm{A}_i) \le \mathrm{Probability}(\mathrm{A}_i) \le \mathrm{Pl}(\mathrm{A}_i)$$

Note that special care must be taken in interpreting the belief of a specific singleton propositional outcome. The statement Bel(A) = 0 can be interpreted as:

(1) After observing a great deal of complex input data (e.g., real-time sensor measurements, multidimensional filtered feature extractions, real-time data base and *a priori* data base information content, natural language text and symbols parsing evidence, quantitative and qualitative communication information, clues, database information) and fusing it all, no information supports the belief Bel(A).

(2) After having made few or no observations, no information supports the belief Bel(A).

Special care must be taken in interpreting the values of the belief function theory variables and valid interpretations can only be done by looking at all the system variables.

3 The Probability Information Content

The probability information content (PIC) variable introduced in [2] assigns a knowledge value to any set of probabilities.

$$PIC \equiv 1 + \frac{\sum_{i=1}^{N} P(i) Log[P(i)]}{Log[N]}$$

A PIC value of one indicates total knowledge is available to make a correct decision. A PIC value of zero indicates that all the hypotheses have an equal probability of occurring and it is not possible to make a good decision.

4 The Inverse Pignistic Probability Postulates

The inverse pignistic probability transform (IPPT) maps the probability set, P(A), of possible outcomes Ω to the basic belief assignments ($\mathrm{Power\text{-}set}(\Omega) = 2^{\Omega}$) representation. There are many representations for such a mapping. A good criteria for such an IPPT mapping would concur with the following postulates:

(1) For a set of uniform probabilities mapped by an IPPT to a set of BBAs, the same set of BBAs must compute a uniform pignistic probability transform.

(2) For any set of probabilities with a PIC value mapped by an IPPT to a set of BBAs and mapped back to pignistic probability, **the PIC value of the pignistic probability must be less than or equal to the original probability PIC value.**

(3) A conservative IPPT mapping would have the most complex distribution of BBAs consistent with the probability distribution (a maximum entropy process i.e., maximizing the sum of all plausibilities).

Mature Data Set

An extreme mapping depends on the maximum maturity of the incomplete information set. This extreme for a system with a mature data set (**MDS**) maximizes the sum of beliefs and minimizes the sum of Plausibilities. This can be simply accomplished by setting the basic belief assignment singleton subset equal to the probabilities and all other subsets equal to zero. This inverse pignistic probability transform maps the probabilities set, P(A), of possible N outcomes Ω to the subset having N singleton elements of the total basic belief assignment $\mathrm{Power\text{-}set}(\Omega) = 2^{\Omega}$ representation. This case sets the beliefs, probabilities, and Plausibilities equal to each other.

$$\begin{aligned} &\mathrm{m}(\mathrm{A}_i) \equiv \mathrm{Probability}(\mathrm{A}_i) \\ &\mathrm{m}(\mathrm{A}_i, \mathrm{A}_j) = \mathrm{m}(\mathrm{A}_i, \mathrm{A}_j, \mathrm{A}_k) = \ldots = 0 \end{aligned}$$

$$\mathrm{Bel}(\mathrm{A}_i) = \mathrm{Probability}(\mathrm{A}_i) = \mathrm{Pl}(\mathrm{A}_i).$$

Using these BBAs, all five [2] pignistic probability transforms calculate the original probability distribution.

Example:

Let the set of probabilities be the uniform distribution $\mathrm{Probability}(\mathrm{A}_i) = \frac{1}{N}$ for $\mathrm{i} = 1,2,..,\mathrm{N}$.

For the **MDS,** the **IPPT** computes the BBAs as:

$$\mathrm{m}(\mathrm{A}_i) \equiv \mathrm{Probability}(\mathrm{A}_i) = \frac{1}{N} \text{ for } \mathrm{i} = 1,2,..,\mathrm{N}.$$

Note that this subset of BBAs, when mapped with all appropriate pignistic probability transforms[2], computes the same uniform pignistic probability distributions:

$$\mathrm{BetP}(\mathrm{A}_i) = \mathrm{PraPl}(\mathrm{A}_i) = \ldots = \mathrm{PrPl}(\mathrm{A}_i) = \frac{1}{N}$$

for $\mathrm{i} = 1,2,..,\mathrm{N}$.

An Ω BBAs Subset

Another inverse pignistic probability transform that maps the probability set, P(A), of possible N outcomes Ω to a subset having N elements of the basic belief assignments $\text{Power-set}(\Omega) = 2^{\Omega}$ is of particular interest. This mapping is accomplished by first ordering the probabilities and then computing the BBAs with the following equations:

$$\{P(A_1) \geq P(A_2) \geq P(A_3) \geq \ldots P(A_{N-1}) \geq P(A_N)\}$$

and computing the N non-zero basic belief assignments as:

$$\begin{aligned}
&m\{A_1, A_2, \ldots, A_{N-1}, A_N\} \equiv N * P(A_N) \\
&m\{A_1, A_2, \ldots, A_{N-2}, A_{N-1}\} \equiv (N-1) * (P(A_{N-1}) - P(A_N)) \\
&m\{A_1, A_2, \ldots, A_{N-3}, A_{N-2}\} \equiv (N-2) * (P(A_{N-2}) - P(A_{N-1})) \\
&m\{A_1, A_2, \ldots, A_{N-3}, A_{N-3}\} \equiv (N-3) * (P(A_{N-3}) - P(A_{N-2})) \\
&\ldots \\
&m\{A_1, A_2, A_3\} \equiv (3) * \left(P(A_3) - P(A_4)\right) \\
&m\{A_1, A_2\} \equiv (2) * \left(P(A_2) - P(A_3)\right) \\
&m\{A_1\} \equiv (1) * \left(P(A_1) - P(A_2)\right)
\end{aligned}$$

Note that the sum of probabilities is one. $P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_{N-1}) + P(A_N) = 1$

The sum of the BBAs is computed as:

$$\begin{aligned}
&\sum m\{A_J\} \equiv N * P(A_N) + (N-1) * (P(A_{N-1}) - P(A_N)) + \\
&(N-1) * (P(A_{N-1}) - P(A_N)) + (N-2) * (P(A_{N-2}) - P(A_{N-1})) + \\
&(N-3) * (P(A_{N-3}) - P(A_{N-2})) + \ldots + (3) * \left(P(A_3) - P(A_4)\right) + \\
&(2) * \left(P(A_2) - P(A_3)\right) + \left(P(A_1) - P(A_2)\right) = \\
&P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_{N-1}) + P(A_N) = 1
\end{aligned}$$

$$\begin{aligned}
\sum_{A_J \in 2^{\Omega}} m(A_J) &= N * P(A_N) + \sum_{k=1}^{N-1} k * (P(A_k) - P(A_{k+1})) \\
&= \sum_{k=1}^{N} P(A_k) = 1
\end{aligned}$$

Therefore the sum of the BBAs are normalized to one.

$$\sum_{A_J \in 2^{\Omega}} m(A_J) = 1$$

Using these BBAs Smets[1] pignistic probability transform will calculate the original probability distribution. The problem with this transform is that other applicable pignistic probability transforms will have a PIC larger than the original probability distribution P(A). This illustrates some of the shortcomings of the original Pignistic Probability transform.

Example:

Let the set of probability be the uniform distribution $\text{Probability}(A_i) = \frac{1}{N}$ for $i = 1,2,..,N$.

The IPPT computes the BBAs as:

$$m(A_1, A_2, \ldots, A_{N-1}, A_N) \equiv 1$$

Note that this subset of BBAs, when mapped with the appropriate pignistic probability transforms, computes the same uniform pignistic probability distributions:

$$\text{BetP}(A_i) = \text{PraPl}(A_i) = \ldots = \text{PrPl}(A_i) = \frac{1}{N}$$

for $i = 1,2,..,N$.

Generalizes Sum Mean:

There are many inverse pignistic probability transforms that have postulate (1). Using a generalized mean equation as a guide in computing the BBAs, an infinite class of IPPT can be computed. Note that the generalized mean equation has the values of s and t equal to each other.

$$M(s,t) = \left(\frac{1}{N} \sum_{i=1}^{N} a_i^s \right)^{\frac{1}{t}}$$

(The arithmetic mean can be computed with a value of s equal to 1 and a value of t equal to 1. The harmonic mean can be computed with a value of s equal to –1 and a value of t equal to -1. The root-mean-square mean can be computed with a value of s equal to 2 and a value of t equal to 2.) The BBAs are computed as:

$$\begin{aligned}
&m(A_3 \mid s,t) = P(A_3)^{\frac{s}{t}} / D \\
&m(A_1, A_2 \mid s,t) = \left(\frac{1}{2} \sum_{i=1}^{2} P(A_i)^s \right)^{\frac{1}{t}} / D \\
&m(A_3, A_4, A_5 \mid s,t) = \left(\frac{1}{3} \sum_{i=3}^{5} P(A_i)^s \right)^{\frac{1}{t}} / D \\
&m(A_1, A_2, \ldots,, A_N \mid s,t) = \left(\frac{1}{N} \sum_{i=1}^{N} P(A_i)^s \right)^{\frac{1}{t}} / D
\end{aligned}$$

Note that D is the normalizing constant of the BBAs.

Example Uniform Distribution:

For the uniform probability distribution $\mathrm{P}(\mathrm{A}_i)=\frac{1}{N}$ with $\mathrm{i}=1,2,..,\mathrm{N}$.

The BBAs are compared as:

$$\mathrm{m}(\mathrm{A}_1,\mathrm{A}_2,...,,\mathrm{A}_\mathrm{N}\mid \mathrm{s},\mathrm{t})=\left(\frac{1}{\mathrm{N}}\sum_{i=1}^{N}\left(\frac{1}{\mathrm{N}}\right)^{s}\right)^{\frac{1}{t}}/D=\frac{1}{N^{\frac{s}{t}}*D}$$

$$\mathrm{m}(\mathrm{A}_1\mid \mathrm{s},\mathrm{t})=\mathrm{m}(\mathrm{A}_2\mid \mathrm{s},\mathrm{t})=\mathrm{m}(\mathrm{A}_1,\mathrm{A}_2\mid \mathrm{s},\mathrm{t})=...=\frac{1}{N^{\frac{s}{t}}*D}$$

Normalizing the basic belief assignments to compute D gives:

$$\sum_{A_J\in\ 2^{\Omega}} m(A_J\mid s,t)=\frac{2^N-1}{N^{\frac{s}{t}}*D}=1$$

with $D=\frac{2^N-1}{N^{\frac{s}{t}}}$.

Substituting back into the BBAs yields

$$\mathrm{m}(\mathrm{A}_1,\mathrm{A}_2,...,,\mathrm{A}_\mathrm{N}\mid \mathrm{s},\mathrm{t})=\frac{1}{2^N-1}$$

and

$$\mathrm{m}(\mathrm{A}_1\mid \mathrm{t})=\mathrm{m}(\mathrm{A}_2\mid \mathrm{t})=\mathrm{m}(\mathrm{A}_1,\mathrm{A}_2\mid \mathrm{t})=...=\frac{1}{2^N-1}\ .$$

Note that any pignistic probability transformation will generate a uniform probability distribution.

Example for s = 1 and t = 1.

For a given probability set $P(A_i)$ with IPPT values of s=1 and t=1, the basic belief assignments are calculated as:

$$\mathrm{m}(\mathrm{A}_\mathrm{i}\mid 1,1)=P(A_i)/D$$

$$\mathrm{m}(\mathrm{A}_\mathrm{i},\mathrm{A}_\mathrm{j}\mid 1,1)=\left(P(A_i)+P(A_j)\right)/(2D)$$

$$\mathrm{m}(\mathrm{A}_\mathrm{i},\mathrm{A}_\mathrm{j},\mathrm{A}_\mathrm{k}\mid 1,1)=\left(P(A_i)+P(A_j)+P(A_k)\right)/(3D)$$

…

$$\mathrm{m}(\mathrm{A}_1,\mathrm{A}_2,...\mathrm{A}_\mathrm{N}\mid 1,1)=\left(P(A_1)+P(A_2)+...+P(A_N)\right)/(N*D)$$

For this case, D is calculated analytically as: $\mathrm{D}=(2^\mathrm{N}-1)/\mathrm{N}$

From [2], The pignistic probability that is proportional to the belief function is calculated as:

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\mathrm{m}(\mathrm{A}_\mathrm{i})+\frac{\mathrm{Bel}(\mathrm{A}_\mathrm{i})}{\mathrm{Bel}(\mathrm{A}_\mathrm{i})+\mathrm{Bel}(\mathrm{A}_\mathrm{j})}\mathrm{m}(\mathrm{A}_\mathrm{i},\mathrm{A}_\mathrm{j})+...$$
$$+\frac{\mathrm{Bel}(\mathrm{A}_\mathrm{i})}{\mathrm{Bel}(\mathrm{A}_\mathrm{i})+\mathrm{Bel}(\mathrm{A}_\mathrm{j})+\mathrm{Bel}(\mathrm{A}_\mathrm{k})}\mathrm{m}(\mathrm{A}_\mathrm{i},\mathrm{A}_\mathrm{j},\mathrm{A}_\mathrm{k})+...$$

Note that, for a singleton element, $Bel(A_i)=m(A_i)$.

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\mathrm{m}(\mathrm{A}_\mathrm{i})+\frac{\mathrm{m}(\mathrm{A}_\mathrm{i})}{\mathrm{m}(\mathrm{A}_\mathrm{i})+\mathrm{m}(\mathrm{A}_\mathrm{j})}\mathrm{m}(\mathrm{A}_\mathrm{i},\mathrm{A}_\mathrm{j})+...$$
$$+\frac{\mathrm{m}(\mathrm{A}_\mathrm{i})}{\mathrm{m}(\mathrm{A}_\mathrm{i})+\mathrm{m}(\mathrm{A}_\mathrm{j})+\mathrm{m}(\mathrm{A}_\mathrm{k})}\mathrm{m}(\mathrm{A}_\mathrm{i},\mathrm{A}_\mathrm{j},\mathrm{A}_\mathrm{k})+...$$

Substituting for the BBAs:

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})}{\mathrm{D}}+\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})/\mathrm{D}}{\mathrm{P}(\mathrm{A}_\mathrm{i})/\mathrm{D}+\mathrm{P}(\mathrm{A}_\mathrm{j})/\mathrm{D}}\left(\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})+\mathrm{P}(\mathrm{A}_\mathrm{j})}{2\mathrm{D}}\right)+...$$
$$+\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})/\mathrm{D}}{\mathrm{P}(\mathrm{A}_\mathrm{i})/\mathrm{D}+\mathrm{P}(\mathrm{A}_\mathrm{j})/\mathrm{D}+\mathrm{P}(\mathrm{A}_\mathrm{k})/\mathrm{D}}\left(\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})+\mathrm{P}(\mathrm{A}_\mathrm{j})+\mathrm{P}(\mathrm{A}_\mathrm{k})}{3\mathrm{D}}\right)+...$$

The terms are of the following form,

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})}{\mathrm{D}}\left(1+\frac{1}{2}+\frac{1}{2}+...+\frac{1}{3}+...\frac{1}{N}\right)$$

Substituting for the D value and enumerating all the terms,

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})\mathrm{N}}{2^\mathrm{N}-1}\left(1+\frac{N-1}{2*1}+\frac{(N-1)(N-2)}{3*2!}+...\frac{1}{N}\right)$$

Rewriting with the summation symbol,

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\frac{\mathrm{P}(\mathrm{A}_\mathrm{i})\mathrm{N}}{2^\mathrm{N}-1}\left(\sum_{i=0}^{N-1}\frac{(N-1)!}{(i+1)*i!(N-1-i)!}\right)$$

Canceling the same terms,

$$\mathrm{PrBl}(\mathrm{A}_\mathrm{i})=\mathrm{P}(\mathrm{A}_\mathrm{i})$$

This shows that since the probabilities, BBAs, and all the pignistic probabilities are all normalized to one[2], the pignistic probability that is **proportional** to the belief function is equal to the original probability.

Note that for a given probability set $P(A_i)$ with IPPT value of s=1 and t=1 used to compute the basic belief assignments (which in turn are used to calculated the pignistic probabilities proportional to the beliefs) both the original probability and the pignistic proportional to belief are equal to each other.

For this same example, the same analogous derivation can be used to show that the self-consistent pignistic probability transform is equal to the original probability distribution.

$$\mathrm{PrScP}(\mathrm{A_i}) = \mathrm{P}(\mathrm{A_i})$$

Generalizes Product Mean:

This section introduces another generalized mean equation to use as a guide for the calculation of another class of BBAs. The other generalized mean equation is:

$$\mu(\mathrm{s,u}) = \left(\prod_{i=1}^{N} a_{\mathrm{i}}^{\mathrm{s}} \right)^{\frac{1}{uN}}$$

(The geometric mean has a s value equal to u.) An infinite class of IPPT can be computed. The BBAs are computed as

$$m(\mathrm{A_3} \mid \mathrm{s,u}) = P(A_3)^{\frac{s}{u}} / D$$
$$m(\mathrm{A_1}, \mathrm{A_2} \mid \mathrm{s,u}) = \left(P(A_1)^s P(A_2)^s \right)^{\frac{1}{2u}} / D$$
$$m(\mathrm{A_3}, \mathrm{A_4}, \mathrm{A_8} \mid \mathrm{s,u}) = \left(P(A_3)^s P(A_4)^s P(A_8)^s \right)^{\frac{1}{3u}} / D$$
$$m(\mathrm{A_1}, \mathrm{A_2}, \ldots,, \mathrm{A_N} \mid \mathrm{s,u}) = \left(\prod_{i=1}^{N} P(A_i)^s \right)^{\frac{1}{uN}} / D$$

Note that D is the normalizing constant of the BBAs.

Example: Probability with Uniform Distribution:

For the probability set of a uniform probability distribution $\mathrm{P}(\mathrm{A}_i) = \frac{1}{N}$ for $\mathrm{i} = 1,2,..,\mathrm{N}$.

$$m(\mathrm{A_1}, \mathrm{A_2}, \ldots,, \mathrm{A_N} \mid \mathrm{s,u}) = \left(\prod_{i=1}^{N} \left(\frac{1}{N} \right)^s \right)^{\frac{1}{uN}} / D = \frac{1}{N^{\frac{s}{u}} * D}$$
$$m(\mathrm{A_1} \mid \mathrm{s,u}) = m(\mathrm{A_2} \mid \mathrm{s,u}) = m(\mathrm{A_1}, \mathrm{A_2} \mid \mathrm{s,u}) = \ldots = \frac{1}{N^{\frac{s}{u}} * D}$$

Normalizing the basic belief assignments to compute D:

$$\sum_{A_J \in 2^\Omega} m(A_J \mid s,u) = \frac{2^N - 1}{N^{\frac{s}{u}} * D} = 1 \text{ with } D = \frac{2^N - 1}{N^{\frac{s}{u}}}.$$

Substituting back into the BBAs gives

$$\mathrm{m}(\mathrm{A_1}, \mathrm{A_2}, \ldots,, \mathrm{A_N} \mid \mathrm{s,u}) = \frac{1}{2^N - 1}$$

and $\mathrm{m}(\mathrm{A_2} \mid \mathrm{s,u}) = \mathrm{m}(\mathrm{A_1}, \mathrm{A_2} \mid \mathrm{s,u}) = \ldots = \frac{1}{2^N - 1}$

Note that any pignistic probability transform will generate a uniform probability distribution.

Choose the inverse pignistic probability transforms via the maximum and minimum functions that have property (1).

$$m(\mathrm{A_3}) = P(A_3) / D$$
$$m(\mathrm{A_1}, \mathrm{A_2}) = Max\left(P(A_1), P(A_2)\right) / D$$
$$m(\mathrm{A_3}, \mathrm{A_4}, \mathrm{A_8}) = Max\left(P(A_3), P(A_4), P(A_8)\right) / D$$
$$m(\mathrm{A_1}, \mathrm{A_2}, \ldots,, \mathrm{A_N}) = Max\left(P(A_1), P(A_2), \ldots, P(A_N)\right) / D$$

Note that D is the normalizing constant of the BBAs.

$$m(\mathrm{A_3}) = P(A_3) / D$$
$$m(\mathrm{A_1}, \mathrm{A_2}) = Min\left(P(A_1), P(A_2)\right) / D$$
$$m(\mathrm{A_3}, \mathrm{A_4}, \mathrm{A_8}) = Min\left(P(A_3), P(A_4), P(A_8),\right) / D$$
$$m(\mathrm{A_1}, \mathrm{A_2}, \ldots,, \mathrm{A_N}) = Min\left(P(A_1), P(A_2), \ldots, P(A_N)\right) / D$$

Note that D is the normalizing constant of the BBAs.

For the uniform probability distribution

$\mathrm{P}(\mathrm{A}_i) = \frac{1}{N}$ with $\mathrm{i} = 1,2,..,\mathrm{N}$.

Both maximum and minimum functions have the following BBAs.

$$m(\mathrm{A_1}, \mathrm{A_2}, \ldots,, \mathrm{A_N}) = \frac{1}{2^N - 1}$$
$$m(\mathrm{A_1}) = m(\mathrm{A_2}) = m(\mathrm{A_1}, \mathrm{A_2}) = \ldots = \frac{1}{2^N - 1}$$

Note that any pignistic probability transform using the above BBAs will generate a uniform probability distribution.

5 Conclusion

For systems where the incomplete information set can be naturally mapped into a belief theory information set and a Bayesian probability theory information set, the need arises to transform either set of fused information into the other set. The mapping of the belief theory fusion results into the probability fusion result set is accomplished via a pignistic probability transform. See [1], [2]. This article introduces a methodology of mapping the fused probability set into the belief function theory results via the inverse pignistic probability transforms.

Three properties of the inverse pignistic probability transform (IPPT) have been identified. (1) For an initial set of uniform probabilities the IPPT must compute a set of BBAs that are used to recompute a uniform pignistic probability transform. (2) For any set of probabilities with a specific PIC value mapped by an IPPT to a set of BBAs and mapped back to pignistic probability, the PIC value of the pignistic probability must be less than or equal to the original probability PIC value. No information can be created by the inverse pignistic probability transform. (3) A conservative IPPT mapping would have a maximum entropy component. This can be accomplished by maximizing the sum of the plausibilities.

An extreme inverse pignistic probability transform has been identified that bound, the BBAs. Also introduced are two classes of equations for computing generalized means. These equations are used as a guide to compute various inverse pignistic probability transforms. These transformations can be used to build and test information fusion systems that address the input of a complex, incomplete and, at times, deceptive information set.

6 References

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