The System Probability Information Content (PIC) Relationship to Contributing Components, Combining Independent Multi-Source Beliefs, Hybrid and Pedigree Pignistic Probabilities

John J. Sudano
Lockheed Martin
Moorestown, NJ, 08057, USA
john.j.sudano@lmco.com

Abstract - *In the design of information fusion systems, the reduction of computational complexity is a key design parameter for real-time implementations. One way to simplify the computations is to decompose the system into subsystems of non-correlated informational components, such as a qualitative informational component, a quantitative informational component, and a complement informational component.*

A probability information content (PIC) variable [1] assigns an information content value to any set of system or sub-system probability distributions. The PIC variable is the normalized entropy computed from the probability distribution. This article derives a PIC variable for a subsystem represented by the complement probabilities. This article also derives a relationship between the PIC variable of sub-system components and the system informational PIC variable.

A hybrid pignistic probability is introduced that is robust in estimating a probability for any maturity of the incomplete data set.

A new methodology of combining independent multi-source beliefs is presented.

A pedigree pignistic probability is introduced that uses some information of the original fused data sets to compute a better pignistic probability.

1 Introduction

A probability information content (PIC) variable assigns an information content value to a probability measure. The PIC variable is the normalized entropy computed from the probability distribution. A PIC value of one indicates total knowledge is available and there is no ambiguity in the decision making; i.e., one of the hypotheses has a probability value of one and the rest have zero. A PIC value of zero indicates that all the hypotheses have an equal probability of occurring and it is not possible to make a good decision.

Given a probability measure P on a set $A=\{a_1,a_2,...,a_N\}$ with respective probabilities $\{P(a_1),P(a_2),...,P(a_N)\}$, the probability information content is:

$$PIC(\mathrm{A}) \equiv 1+\frac{\sum_{i=1}^{N} P(a_i)Log[P(a_i)]}{Log[N]} \quad (1)$$

In the design of information fusion systems, the reduction of computational complexity is a key design parameter for real-time implementations. One way to simplify the computations is to decompose the incomplete system information set into subsystems of uncorrelated informational components.

The subsystems of uncorrelated informational components can be labeled as a mature [1] or non-mature data set. The Pignistic probabilities proportional to beliefs (PrBl) and self-consistent pignistic probability (PrScP) are valid only for a mature data set. Smets pignistic probability BetP [2], the pignistic probability proportional to all the plausibilities (PraPl), the pignistic probability proportional to the plausibilities (PrPl), and a new introduced hybrid pignistic probability (PrHyb) are robust for all type of data sets.

Some N subsystems of uncorrelated informational components can be represented by N sets $\{\mathrm{m}_1,\mathrm{m}_2,...,\mathrm{m}_\mathrm{N}\}$ of basic belief assignments (BBAs), a new methodology is presented for combining or fusing the BBAs to produce one set of BBA $\{\mathrm{m}_{1,2,...\mathrm{N}}\}$.

A Pedigree Pignistic Probability (PrPed) is introduced that uses the fused BBA with the pedigree information of each subsystems to compute a better pignistic probability.

2 Complement Representation PIC

The complement A_i^c of an event A_i is the set of all outcomes in the sample space that are not included in the outcomes of event A_i. If the probability of event

A_i occurring is $P(A_i)$ then the probability of its complement is:

$$P(A_i^C) = 1 - P(A_i) \cdot \qquad (2)$$

Since the sum of the singleton probabilities is one,

$$\sum_{i=1}^{N} P(A_i) = 1 , \qquad (3)$$

It follows that the sum of the probability of the singleton complement is N-1:

$$\sum_{i=1}^{N} P(A_i^C) = \sum_{i=1}^{N} 1 - P(A_i) = N - 1 \cdot \qquad (4)$$

The probability information content for a system represented by complement probabilities is:

$$PIC(\mathrm{A}^C) \equiv 1 + \frac{\sum_{i=1}^{N} (1 - P(A_i^C)) Log\left[1 - P(A_i^C)\right]}{Log[N]} \qquad (5)$$

A normalized complement probability is defined as:

$$P(\hat{A}_i^C) \equiv \frac{P(A_i^C)}{N-1} = \frac{1 - P(A_i)}{N-1}$$
(6)

The probability information content for a system represented by normalized complement probabilities is:

$$PIC(\hat{A}^C) \equiv 1 + \frac{\sum_{i=1}^{N} \left(1 - (N-1) P(\hat{A}_i^C)\right) Log\left[1 - (N-1) P(\hat{A}_i^C)\right]}{Log[N]} \qquad (7)$$

3 Mutually Exclusive Subsystem Disjoint Decomposition

In the design of information fusion systems, the reduction of computational complexity is a key design parameter for real-time implementations. One way to simplify the computations is to decompose the system into subsystems of mutually exclusive disjoined components.

Let Ω be the set of possible outcomes, where the outcomes are mutually exclusive and exhaustive singleton elements of the decision environment. In some systems the Ω set is decomposed into mutually exclusive subsystems $\Omega = \Omega_I \oplus \Omega_J \oplus \Omega_K \oplus \ldots$. For systems [1] with a complex input (e.g., real-time sensor measurements, multidimensional filtered feature extractions, real-time data base and *a priori* data base information content, real-time natural language text and symbols parsing evidence, quantitative and qualitative communication clues, and inconsistent errors), a power set of the outcomes is a better representation of the incomplete information set.

$$\text{Power - set } (\Omega_I \oplus \Omega_J \oplus \Omega_K \oplus \ldots) = 2^{\Omega_I} \otimes 2^{\Omega_J} \otimes 2^{\Omega_K} \otimes \ldots \qquad (8)$$

Note that this power set is much smaller than the full system power set, for two or more subsystems.

$$2^{\Omega} \geq 2^{\Omega_I} \otimes 2^{\Omega_J} \otimes 2^{\Omega_K} \otimes \ldots \qquad (9)$$

For the subsystems supporting the number I, J, K,… of possible hypothesis with respective components i, j, k,… , the sub-system PIC values are computed from the individual probabilities:

$$PIC(P_I) \equiv 1 + \frac{\sum_{i=1}^{I} P(i) Log[P(i)]}{Log[I]},$$
$$PIC(P_J) \equiv 1 + \frac{\sum_{j=1}^{J} P(j) Log[P(j)]}{Log[J]}, \qquad (10)$$
$$PIC(P)_K \equiv 1 + \frac{\sum_{i=1}^{K} P(i) Log[P(i)]}{Log[K]}$$

The system probability can be computed from the uncorrelated subsystem probability distributions as:

$$P(s) \equiv P(i) * P(j) * P(k) * \ldots \qquad (11)$$

Note that the system probability is normalized to one provided that the subsystem probability distributions are normalized to one.

$$\sum_{s=1}^{I*J*K*\ldots} P(s) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \ldots . P(i) P(j) P(k) \ldots$$
$$\sum_{s=1}^{I*J*K*\ldots} P(s) = \sum_{i=1}^{I} P(i) \sum_{j=1}^{J} P(j) \sum_{k=1}^{K} . P(k) \ldots \qquad (12)$$
$$\sum_{s=1}^{I*J*K*\ldots} P(s) = 1$$

The system PIC variable is:

$$PIC(P_S) \equiv 1 + \frac{\sum_{s=1}^{I*J*K*\ldots} P(s) Log[P(s)]}{Log[I * J * K \ldots]} \qquad (13)$$

Substituting the system probability in terms of the subsystem probabilities:

$$PIC(P_S) = 1 + \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \ldots (P(i) P(j) P(k) \ldots) Log[(P(i) P(j) P(k) \ldots)]}{L \ldots I * J * K}$$

(14)

Expanding the logarithmic term:

$$PIC(P_S) = 1 + \frac{\sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} ....(P(i)P(j)P(k)...)\left(Log[P(i)] + Log[P(j)] + Log[P(k)] + ...\right)}{Log[I] \ + \ Log[J] \ + \ Log[K] \ + \ ...} \quad (15)$$

Summing over the solitary probability terms:

$$PIC(P_S) = 1 + \frac{\left(\sum_{i=1}^{I} P(i)Log[P(i)]\right) + \left(\sum_{j=1}^{J} P(j)Log[P(j)]\right) + \left(\sum_{k=1}^{K} P(k)Log[P(k)]\right) + (...)}{Log[I] \ + \ Log[J] \ + \ Log[K] \ + \ ...} \quad (16)$$

Substituting the subsystem PIC variables:

$$PIC(P_S) = 1 + \frac{(PIC(P_I) - 1)Log[I] + (PIC(P_J) - 1)Log[J] + (PIC(P_K) - 1)Log[K] + ...}{Log[I] \ + \ Log[J] \ + \ Log[K] \ + \ ...} \quad (17)$$

Simplifying the logarithmic coefficients shows that the system PIC is a weighted average of the subsystem PICs:

$$PIC(P_S) = \frac{PIC(P_I)Log[I] + PIC(P_J)Log[J] + PIC(P_K)Log[K] + ...}{Log[I] \ + \ Log[J] \ + \ Log[K] \ + \ ...} \quad (18)$$

Note that if all the subsystem information sets support the same number of hypotheses (i.e., I = J = K =…), then the system PIC equation simplifies to the average of the subsystem PICs.

$$PIC(P_S) \ = \ \frac{PIC(P_I) + PIC(P_J) + PIC(P_K) + ...}{1 \ + \ 1 \ + \ 1 \ + \ ...} \quad (19)$$

4 Simplifying Notation

In order to simplify the derivations, the [1] compound-to-sum of singletons operator is used. The compound-to-sum operator, $\vec{C}^S$, on singleton elements has the following properties:

$$\vec{C}^S[Bel(\{A_j, A_k, ..., A_z\})] \equiv Bel(A_j) + Bel(A_k) + ... + Bel(A_z)$$
$$\vec{C}^S[Pl(\{A_j, A_k, ..., A_z\})] \equiv Pl(A_j) + Pl(A_k) + ... + Pl(A_z)$$
$$\vec{C}^S[m(\{A_j, A_k, ..., A_z\})] \equiv m(A_j) + m(A_k) + ... + m(A_z) \quad (20)$$

5 Hybrid Pignistic Probability

A series of Pignistic probabilities were introduced in [1] that use some *Posteriori* information to proportionally decompose the compound basic belief assignments into Pignistic probabilities. The Pignistic probability proportional to all plausibilities (PraPl) is computed using the following equation.

$$\mathrm{PraPl}(A_i) \equiv \mathrm{Bel}(A_i) + \varepsilon \ \mathrm{Pl}(A_i) \quad (21)$$

with $$\varepsilon \ = \frac{1 - \sum_{A_i \subseteq \Omega} \mathrm{Bel}(A_i)}{\sum_{A_i \subseteq \Omega} \mathrm{Pl}(A_i)} \quad (22)$$

The hybrid pignistic probability transformation distributes the basic belief assignments proportionally to PraPl among each singleton element of $A_i \subseteq A_J$ with $A_i \subseteq \Omega$ for all $A_J \in 2^\Omega$.

$$\mathrm{PrHyb}\,(A_i) = \sum_{A_M \supseteq A_i} \left(\frac{\mathrm{PraPl}\,(A_i)}{\vec{C}^S[\mathrm{PraPl}\,(A_M)]} \right) m(A_M) \quad (23)$$

The pignistic probability is normalized to one.

$$\sum_{A_i \subseteq \Omega} \mathrm{PrHyb}(A_i) = 1 \quad (24)$$

For each singleton element A_i, the pignistic probability is bound between the Belief and the Plausibility.

$$\mathrm{Bel}(A_i) \le \mathrm{PrHyb}(A_i) \le \mathrm{Pl}(A_i) \quad (25)$$

6 Discussion

In complex decision making, the incomplete information set can be divided into independent components and each component can be labeled as a mature [1] or non-mature data set. The Pignistic probabilities proportional to beliefs (PrBl) and self-consistent pignistic probability (PrScP) are valid only for a mature data set. Smets pignistic probability BetP [2], the pignistic probability proportional to all the plausibilities (PraPl), the pignistic probability proportional to the plausibilities (PrPl), and hybrid pignistic probability (PrHyb) are robust for all type of data sets.

Some of these pignistic probabilities are described by the probability proportionally function ρ,

$$\rho_J = \{|\mathrm{A_J}|, \ \mathrm{Pl_J}, \ \mathrm{PraPl_J}, \ \mathrm{Bel_J}\} \quad (26)$$

with the corresponding pignistic probabilities.

$$\Pi_J = \{\mathrm{BetP_J}, \ \mathrm{PrPl_J}, \ \mathrm{PrHyb_J}, \ \mathrm{PrBl_J}\} \quad (27)$$

7 A New Methodology of Combining Independent Multi-Source Beliefs

In some systems the incomplete information set can be divided into N independent component sets $\{m_1, m_2, ..., m_N\}$ of basic belief assignments (BBAs) and M sets of probabilities. This section introduces a new methodology of combining the BBAs.

$$m_{1,2,...,N}(A) = \sum_{B \cap C \cap ... \cap Z = A} m_1(B) m_2(C) ... m_N(Z) \left(\frac{f(A;B,C,...,Z)}{\Delta} \right)$$

$$f(A;B,C,...,Z) = \left(\frac{\vec{C}^S[\rho_1(A)]}{\vec{C}^S[\rho_1(B)]} \right) \left(\frac{\vec{C}^S[\rho_2(A)]}{\vec{C}^S[\rho_2(C)]} \right) .. \left(\frac{\vec{C}^S[\rho_N(A)]}{\vec{C}^S[\rho_N(Z)]} \right) \quad (28)$$

Δ is the normalizing factor of summing the BBA to one.

$$\sum_{A \in 2^\Omega} m_{1,2,...,N}(A) = 1 \quad (29)$$

The combined BBAs can be used with an appropriate pignistic probability transform to make good decisions.

Note that in the combination or fusion process of the BBAs, the pedigree information of each set of BBAs is lost. Saving the probability proportionally function for each member of the singleton set $\rho_1, \rho_2, ..., \rho_N$ can be used to compute the pignistic probability transform.

8 Pedigree Pignistic Probability

The Pedigree Pignistic Probability (PrPed) is introduced that uses the fused BBAs with the probability proportionally functions, $\rho_1, \rho_2, ..., \rho_N$, to compute a better pignistic probability estimate, a Bayesian equivalent.

The Pedigree Pignistic Probability is computed for each singleton element of $A \subseteq B$ with $A \subseteq \Omega$ for all $B \in 2^\Omega$.

$$\text{PrPed}\ (C) = \sum_{C \subseteq A} m_{1,2,...,N}(A) \left(\frac{g(C;A)}{\Delta} \right)$$

$$g(C;A) = \left(\frac{\vec{C}^S[\rho_1(C)]}{\vec{C}^S[\rho_1(A)]} \right) \left(\frac{\vec{C}^S[\rho_2(C)]}{\vec{C}^S[\rho_2(A)]} \right) .. \left(\frac{\vec{C}^S[\rho_N(C)]}{\vec{C}^S[\rho_N(A)]} \right) \quad (30)$$

Δ is the normalizing factor of summing the Pignistic probability to one.

9 A Cardinality Simplification

The methodology of belief combination and the pedigree pignistic probability simplifies significantly when used in conjunction with probability proportionally function of the cardinality value of the power set member.

$$m_{1,2,...,N}(A; cardinality) = \sum_{B \cap C \cap ... \cap Z = A} m_1(B) m_2(C) ... m_N(Z) \left(\frac{f(A;B,C,...,Z)}{\Delta} \right)$$

$$f(A;B,C,...,Z) = \left(\frac{|A|}{|B|} \right) \left(\frac{|A|}{|C|} \right) .. \left(\frac{|A|}{|Z|} \right) \quad (31)$$

Substituting one equation into the other:

$$m_{1,2,...,N}(A; cardinality) = \sum_{B \cap C \cap ... \cap Z = A} m_1(B) m_2(C) ... m_N(Z) \left(\frac{1}{\Delta} \right) \left(\frac{|A|^N}{|B||C|..|Z|} \right) \quad (32)$$

The Pedigree Pignistic Probability is:

$$\text{PrPed}\ (C; cardinality) = \sum_{C \subseteq A} m_{1,2,...,N}(A; cardinality) \left(\frac{g(C;A)}{\Delta} \right)$$

$$g(C;A) = \left(\frac{|C|}{|A|} \right) \left(\frac{|C|}{|A|} \right) .. \left(\frac{|C|}{|A|} \right) \quad (33)$$

Substituting one equation into the other:

$$\text{PrPed}\ (C; cardinality) = \sum_{C \subseteq A} m_{1,2,...,N}(A; cardinality) \left(\frac{|C|}{|A|} \right)^N \quad (34)$$

Since C is a singleton member its cardinality is one. i.e., $|C| = 1$

$$\text{PrPed}\ (C; cardinality) = \sum_{C \subseteq A} m_{1,2,...,N}(A; cardinality) \frac{1}{|A|^N} \quad (35)$$

Defining a new set of the BBAs as:

$$m^*_{1,2,...,N}(A) = \frac{m_{1,2,...,N}(A; cardinality)}{|A|^{N-1}} \quad (36)$$

$$m^*_{1,2,...,N}(A) = \sum_{B \cap C \cap ... \cap Z = A} m_1(B) m_2(C) ... m_N(Z) \left(\frac{1}{\Delta} \right) \left(\frac{|A|}{|B||C|..|Z|} \right) \quad (37)$$

The Pedigree Pignistic Probability for the new set of BBAs becomes:

$$\text{PrPed}\ (C; cardinality) = \quad (38)$$

$$= \sum_{C \subseteq A} m_{1,2,...,N}(A; cardinality) \frac{1}{|A|^N} = \sum_{C \subseteq A} \frac{m^*_{1,2,...,N}(A)}{|A|}$$

Calculating Smets pignistic probability for the new set of BBA.

$$\text{BetP}\ (C) = \sum_{C \subseteq A} \frac{m^{*}_{1,2,\ldots,N}(A)}{|A|} \tag{39}$$

For this new set of redefined BBAs the Pedigree pignistic probability and Smets pignistic probability are equivalent.

$$\text{PrPed}\ (\text{C}; cardinality) = \text{BetP}\ (C) \tag{40}$$

In literature, Fister and Mitchell [3] address the combination of two belief subsystems:

$$m^{*}_{12}(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{|B \cap C|}{\Delta\ |B|\ |C|} \right) = \tag{41}$$

$$= \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{|A|}{\Delta\ |B|\ |C|} \right)$$

Δ is the normalizing factor of summing the BBA to one and $|B|$ is the cardinality of B.

10 Example:

The following example illustrates some design concepts presented in this article. A set of the m BBAs is given as:

m0(1,0,0,0) = 0.16
m0(0,1,0,0) = 0.14
m0(0,0,1,0) = 0.01
m0(0,0,0,1) = 0.02
m0(1,1,0,0) = 0.20
m0(1,0,1,0) = 0.09
m0(1,0,0,1) = 0.04
m0(0,1,1,0) = 0.04
m0(0,1,0,1) = 0.02
m0(0,0,1,1) = 0.01
m0(1,1,1,0) = 0.10
m0(1,1,0,1) = 0.03
m0(1,0,1,1) = 0.03
m0(0,1,1,1) = 0.03
m0(1,1,1,1) = 0.08 (42)

The beliefs are calculated as:

$$Bel\_m0 = (0.16,\ .14,\ .01,\ .02) \tag{43}$$

For these BBAs the Plausibilities are calculated to be:

$$Pl\_m0 = (0.73,\ .64,\ .39,\ .26) \tag{44}$$

The Smets pignistic probabilities are computed from the above BBAs as:

$$BetP\_m0 = (0.398,\ .343,\ .153,\ .105) \tag{45}$$

The pignistic probability proportional to all Plausibilities (PraPl) is computed as:

$$\text{PraPl\_m0} = (0.402,\ .352,\ .139,\ .106)$$
with $\varepsilon = 0.331683$ (46)

Let another set of the m1 BBAs be:

m1(1,0,0,0) = 0.05
m1(0,1,0,0) = 0.00
m1(0,0,1,0) = 0.00
m1(0,0,0,1) = 0.00
m1(1,1,0,0) = 0.39
m1(1,0,1,0) = 0.19
m1(1,0,0,1) = 0.18
m1(0,1,1,0) = 0.04
m1(0,1,0,1) = 0.02
m1(0,0,1,1) = 0.01
m1(1,1,1,0) = 0.04
m1(1,1,0,1) = 0.02
m1(1,0,1,1) = 0.03
m1(0,1,1,1) = 0.03
m1(1,1,1,1) = 0.00 (47)

The beliefs are calculated as:

$$Bel\_m1 = (0.05,\ .00,\ .00,\ .00) \tag{48}$$

For these BBAs the Plausibilities are calculated to be:

$$Pl\_m1 = (0.90,\ .54,\ .34,\ .29) \tag{49}$$

Computing the pignistic probability proportional to all plausibilities.

with $$\varepsilon = \frac{1 - \sum_{A_i \subseteq \Omega} \text{Bel\_m1}(\text{A}_i)}{\sum_{A_i \subseteq \Omega} \text{Pl\_m1}(\text{A}_i)} = .458937 \tag{50}$$

$$\text{PraPl}(\text{A}_i) \equiv \text{Bel}(\text{A}_i) + \varepsilon\ \text{Pl}(\text{A}_i) \tag{51}$$

$$\text{Pr}\,aPl\_m1 = (0.463, 0.248, 0.156, 0.133) \tag{52}$$

A set of the m2 BBAs is given as:

$$m2(1,0,0,0) = 0.46$$
$$m2(0,1,0,0) = 0.14$$
$$m2(0,0,1,0) = 0.01$$
$$m2(0,0,0,1) = 0.02$$
$$m2(1,1,0,0) = 0.21$$
$$m2(1,0,1,0) = 0.09$$
$$m2(1,0,0,1) = 0.04$$
$$m2(0,1,1,0) = 0.01$$
$$m2(0,1,0,1) = 0.02$$
$$m2(0,0,1,1) = 0.00$$
$$m2(1,1,1,0) = 0.00$$
$$m2(1,1,0,1) = 0.00$$
$$m2(1,0,1,1) = 0.00$$
$$m2(0,1,1,1) = 0.00$$
$$m2(1,1,1,1) = 0.00 \quad (53)$$

The beliefs are calculated as:

$$Bel\_m2 = (0.46,\ .14,\ .01,\ .02) \quad (54)$$

For these BBAs the Plausibilities are calculated to be:

$$Pl\_m2 = (0.80,\ .38,\ .11,\ .08) \quad (55)$$

11 Cardinality Weighting

$$m_{0,1,2,}(A; cardinality) = \sum_{B\cap C\cap ...\cap Z = A} m\ (B) m_1(C) m_3(D) \left(\frac{1}{\Delta}\right)\left(\frac{|A|^3}{|B||C||D|}\right) \quad (56)$$

$$m_{012}(1,0,0,0; cardinality) = 0.6656$$
$$m_{012}(0,1,0,0; cardinality) = 0.1141$$
$$m_{012}(0,0,1,0; cardinality) = 0.0049$$
$$m_{012}(0,0,0,1; cardinality) = 0.0033$$
$$m_{012}(1,1,0,0; cardinality) = 0.1768$$
$$m_{012}(1,0,1,0; cardinality) = 0.0277$$
$$m_{012}(1,0,0,1; cardinality) = 0.0061$$
$$m_{012}(0,1,1,0; cardinality) = 0.0009$$
$$m_{012}(0,1,0,1; cardinality) = 0.0006$$
$$m_{012}(0,0,1,1; cardinality) = 0.0000$$
$$m_{012}(1,1,1,0; cardinality) = 0.0000$$
$$m_{012}(1,1,0,1; cardinality) = 0.0000$$
$$m_{012}(1,0,1,1; cardinality) = 0.0000$$
$$m_{012}(0,1,1,1; cardinality) = 0.0000$$
$$m_{012}(1,1,1,1; cardinality) = 0.0000 \quad (57)$$

The Pedigree Pignistic Probability is computed with the following equation:

$$\text{PrPed}\ (C; cardinality) = \sum_{C\subseteq A} m_{1,2,...,N}(A; cardinality)\frac{1}{|A|^N} \quad (58)$$

$$\text{PrPed}\ (cardinality) = (0.8227,\ .1622,\ .0101,\ .0049) \quad (59)$$

Defining a new set of the BBAs as:

$$m^*_{1,2,...,N}(A) = \sum_{B\cap C\cap ...\cap Z = A} m_1(B) m_2(C) ... m_N(Z)\left(\frac{1}{\Delta}\right)\left(\frac{|A|}{|B||C|...|Z|}\right) \quad (60)$$

$$m^*(1,0,0,0) = 0.7915$$
$$m^*(0,1,0,0) = 0.1357$$
$$m^*(0,0,1,0) = 0.0058$$
$$m^*(0,0,0,1) = 0.0039$$
$$m^*(1,1,0,0) = 0.0526$$
$$m^*(1,0,1,0) = 0.0082$$
$$m^*(1,0,0,1) = 0.0018$$
$$m^*(0,1,1,0) = 0.0003$$
$$m^*(0,1,0,1) = 0.0002$$
$$m^*(0,0,1,1) = 0.0000$$
$$m^*(1,1,1,0) = 0.0000$$
$$m^*(1,1,0,1) = 0.0000$$
$$m^*(1,0,1,1) = 0.0000$$
$$m^*(0,1,1,1) = 0.0000 \quad (61)$$
$$m^*(1,1,1,1) = 0.0000$$

Calculating Smets pignistic probability for the new set of BBA.

$$\text{BetP}\ (C) = \sum_{C\subseteq A} \frac{m^*_{1,2,...,N}(A)}{|A|} \quad (62)$$

$$\text{BetP}\ = (0.8227,\ .1622,\ .0101,\ .0049) \quad (63)$$

The Pedigree pignistic probability and Smets pignistic probability are the same.

$$\text{PrPed}\ (cardinality) = \text{BetP} \quad (64)$$

Note that by using a *priori* information inherent in the cardinality probability proportionally function the smaller probabilities tend to be overestimated while the larger probabilities tend to be underestimated.

12 A Mixed Weighting

For this example the mixed weighting described by the following (belief, hybrid, belief; BHB) probability proportionally function:

$$\rho = \{\text{Bel\_m0}, \ \text{PraPl\_m1}, \ \text{Bel\_m2}\} \qquad (65)$$

$$m_{0,1,2}(A;BHB) = \sum_{B\cap C\cap D=A} m_0(B)m_1(C)m_2(D)\left(\frac{f(A;B,C,D)}{\Delta}\right)$$

$$f(A;B,C,D) = \left(\frac{\vec{C}^S[Bel\_m0(A)]}{\vec{C}^S[Bel\_m0(B)]}\right)\left(\frac{\vec{C}^S[\Pr aPl\_m1(A)]}{\vec{C}^S[\Pr aPl\_m1(C)]}\right)\left(\frac{\vec{C}^S[Bel\_m2(A)]}{\vec{C}^S[Bel\_m2(D)]}\right)$$

(66)

$$\begin{aligned}
\text{m}_{012}(1,0,0,0;BHB) &= 0.79709 \\
\text{m}_{012}(0,1,0,0;BHB) &= 0.05615 \\
\text{m}_{012}(0,0,1,0;BHB) &= 0.00011 \\
\text{m}_{012}(0,0,0,1;BHB) &= 0.00026 \\
\text{m}_{012}(1,1,0,0;BHB) &= 0.12580 \\
\text{m}_{012}(1,0,1,0;BHB) &= 0.01597 \\
\text{m}_{012}(1,0,0,1;BHB) &= 0.00385 \\
\text{m}_{012}(0,1,1,0;BHB) &= 0.00042 \\
\text{m}_{012}(0,1,0,1;BHB) &= 0.00035 \\
\text{m}_{012}(0,0,1,1;BHB) &= 0.0000 \\
\text{m}_{012}(1,1,1,0;BHB) &= 0.0000 \\
\text{m}_{012}(1,1,0,1;BHB) &= 0.0000 \\
\text{m}_{012}(1,0,1,1;BHB) &= 0.0000 \\
\text{m}_{012}(0,1,1,1;BHB) &= 0.0000 \\
\text{m}_{012}(1,1,1,1;BHB) &= 0.0000 \qquad (67)
\end{aligned}$$

The Pedigree Pignistic Probability is computed with the following equation:

$$\text{PrPed}\ (C;BHB) = \sum_{C\subseteq A} m_{012}(A)\left(\frac{g(C;A)}{\Delta}\right) \qquad (68)$$

$$g(C;A) = \left(\frac{\vec{C}^S[Bel\_m0(C)]}{\vec{C}^S[Bel\_m0(A)]}\right)\left(\frac{\vec{C}^S[\Pr aPl\_m1(C)]}{\vec{C}^S[\Pr aPl\_m1(A)]}\right)\left(\frac{\vec{C}^S[Bel\_m2(C)]}{\vec{C}^S[Bel\_m2(A)]}\right)$$

(69)

$$\text{PrPed(BHB)} = (0.93188,\ .06770,\ .00013,\ .00029)$$

(70)

Note that by using some *posteriori* information inherent in the probability proportionally function better estimates of the pignistic probabilities are obtained.

$$\text{PrPed}\ (cardinality) = (0.8227,\ .1622,\ .0101,\ .0049)$$

(71)

13 Conclusion:

In the design of information fusion systems, the reduction of computational complexity is a key design parameter for real-time implementations. One method of obtaining some simplification of the computations is to decompose the system into subsystems of mutually exclusive disjoined informational components.

This article derived a simple relationship between the total system informational PIC variable and the weighted average of disjoint decomposition sub-system components PICs.

The probability information content for a system represented by complement probabilities was demonstrated.

A hybrid pignistic probability was introduced that is robust in estimating a probability for any maturity of the incomplete data set.

A new methodology of combining independent multi-source beliefs was presented.

A pedigree pignistic probability was introduced that use some information of the original fused data sets to compute a better pignistic probability.

References:

[1] Sudano, John, J., "Pignistic Probability Transforms for Mixes of Low- and High- Probability Events," 4th International Conference on Information Fusion 2001, Montreal, QC, Canada, August 2001, pages TUB3 23-27.

[2] Smets P., Kennes, R., "The Transferable Belief Model," Artificial Intelligence, vol. 66, pages 191-243, 1994.

[3] Fister, T., Mitchell, R., "Modified Dempster-Shafer with Entropy Based Belief Body Compression", Proc. 1994 Joint Service Combat Identification Systems Conference (CISC), Naval Postgraduate School, CA, August 1994, pp. 281-310