Equivalence Between Belief Theories and Naïve Bayesian Fusion for Systems with Independent Evidential Data: Part I, The Theory

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Abstract – *The process of fusing multiple independent sensor measurements, communication link data from other independent systems, and dynamic data base information is essential to support critical decisions in a timely way. Many real systems can be mapped to such a process. The independence of the input evidential data with an equal probable uniform prior probability distribution (i.e., Naïve Bayesian fusion) greatly simplifies the mathematical techniques used to properly fuse the evidential data. Equivalence between Belief Fusion and Naïve Bayesian is shown for this process. The equivalence comparison is done in probability space. The title of a 2001 colloquium* ***, "Data Fusion & Target ID: Dempster-Shafer & Probability Theories Holy War," depicts the state of mind of many researchers. The goal of this article is to show that large areas from both mathematical camps are equivalent.*** *This equivalence can be exploited by reducing the computational complexity of the fusion process. The fusion can be done in the linear probability set space rather than the exponential power-set representation of the belief space.*

For a system with 10 possible hypotheses, The fusion of independent data in belief space would involve the fusion of as many as 1024 members of the power set, while exactly the same results can be obtained by fusion of 10 members in probability space. This implies a non-trivial saving in computation complexity for the implementation of many real systems, such as medical diagnostic systems, oil exploration systems, combat identification, ballistic missile component discrimination, and homeland security automated systems.

Keywords: Pignistic Probability, Belief Fusion, Probability Fusion, Belief Probability Equivalence.

1. Introduction –

Many mathematical techniques (Belief Theory [1], DSmT [12], Bayesian Networks [14],…) have evolved to address real world information fusion processes. Many of these techniques have been successful over a narrow band of physical attribute space. The goal is to better understand the mathematical techniques so that they can be properly used to design robust systems over the full physical range of physical attribute space. The objective of this paper is to bring some understanding between the world of probability and belief fusion.

2. Background

Let Ω be the set of possible outcomes [9], where the outcomes are mutually exclusive and exhaustive singleton elements of the decision environment. In Bayesian formalism, the probabilities are assigned only to the singleton subsets of the quantitative information set. These probabilities are used to make the decisions. For systems with a complex input (real-time sensor measurements, multidimensional filtered feature extractions, real-time data base and *a priori* data base information content, natural language text and symbols parsing evidence, quantitative and qualitative communication clues, and inconsistent errors), a $\text{Power-set}(\Omega) = 2^{\Omega}$ representation of the outcomes and a two-level (lower & upper) probability portrayal is a better representation of the incomplete information set; i.e., some sensor measurements of attributes populate more than one hypothesis. Belief theories maintain a two-level probabilistic portrayal of the information set: the Belief or credal level and the Plausibility level. The primary foundation in any decision proposition A_i is the value of the Belief Bel(A_i), while the plausibility $Pl(A_i)$ provides the secondary support for the decision. The basic belief assignment (BBA) $\text{m}(\text{A}_J)$ represents the strength of all the incomplete information set for the outcome A_J. The assignments of BBAs values to subsets $A_J \in 2^{\Omega}$ are constrained by the normalization constraint equation.

$$\sum_{A_J \in 2^{\Omega}} m(A_J) = 1 \qquad (1)$$

Using the BBAs as the representative of the incomplete information set, the Belief function can be computed. The Belief of A_J is the sum of all $m(A_K)$ for subsets A_K contained in A_J :

$$Bel(A_J) = \sum_{A_K \supseteq A_J} m(A_K) \qquad (2)$$

The Plausibility of the subset A_J is the sum $m(A_K)$ for all subsets A_K that have a non-null intersection with A_J. It is given by:

$$Pl(A_J) = \sum_{A_K \cap A_J \neq 0} m(A_K) \quad (3)$$

A useful notation to simplify some derivations is the compound-to-sum operator, $\vec{C}^S$ [6], which operates on singleton elements with the following properties:

$$\vec{C}^S[Bel(\{A_j, A_k, ..., A_z)] \equiv Bel(A_j) + Bel(A_k) + ... + Bel(A_z)$$
$$\vec{C}^S[Pl(\{A_j, A_k, ..., A_z\})] \equiv Pl(A_j) + Pl(A_k) + ... + Pl(A_z)$$
$$\vec{C}^S[m(\{A_j, A_k, ..., A_z\})] \equiv m(A_j) + m(A_k) + ... + m(A_z) \quad (4)$$

The generalized belief fusion algorithm [10,13] depends on probability proportionality weighting functions ρ. The choice of this function is dictated by the nature of the individual belief data set. For any belief data set, ρ can be chosen to be:

$$\rho_J = \{\ 1,\ |\mathrm{A_J}|,\ \mathrm{Pl_J},\ \mathrm{PraPl_J}\ \} \quad (5)$$

with the constant "1" , the cardinality of *A*, |*A*|, the plausibility, Pl, and proportional to all plausibilities, PraPl. Special care must be used for $\rho = \{\ Bel\ \}$ since it may give erroneous results if used with a non-mature belief data set. Therefore, the probability proportionality function ρ weighting functions can have the following functional values:

$$\rho_J = \{\ 1,\ |\mathrm{A_J}|,\ \mathrm{Pl_J},\ \mathrm{PraPl_J},\ \mathrm{Bel_J}\} \quad (6)$$

With the corresponding Pignistic Probability Transform [9,13]:

$$\Pi_J = \{\mathrm{PrNPl},\ \mathrm{BetP_J},\ \mathrm{PrPl_J},\ \mathrm{PrHyb_J},\ \mathrm{PrBl_J}\} \quad (7)$$

The pignistic probability transforms distribute the basic belief assignments proportionally to ρ among each single-ton element of $C' \subseteq A$ with $C' \subseteq \Omega$ for all $A \in 2^{\Omega}$.

$$\Pi_J(C') = \sum_{A \supseteq C'} \left(\frac{\rho_J(C')}{\vec{C}^S[\rho_J(A)]} \right) m(A) \quad (8)$$

An example of the Pignistic Probability used to estimate the probability is the pignistic probability proportional to normalized plausibility (PrNPl) [13] which has a $\rho = \{1\}$. It is computed for each singleton element of $C \subseteq A$ with $C \subseteq \Omega$ for all $A \in 2^{\Omega}$.

$$\mathrm{PrNPl}\ (C) = \sum_{C \cap A \neq 0} m(A) \left(\frac{1}{\Delta} \right) \quad (9)$$

Δ is the normalizing factor of summing the PrNPl to one.

$$\sum_{C \subseteq \Omega} \mathrm{PrNPl}\ (C) = 1 \quad (10)$$

The Pedigree Pignistic Probability (PrPed) introduced in [10] uses the fused BBAs with the probability proportionally functions, $\rho_1, \rho_2, ..., \rho_N$, to compute a better pignistic probability estimate when used in conjunction with the generalized belief fusion algorithm. The Pedigree Pignistic Probability is computed for each singleton element of $C \subseteq A$ with $C \subseteq \Omega$ for all $A \in 2^{\Omega}$.

$$\mathrm{PrPed}\ (C) = \sum_{C \subseteq A} m_{1,2,...,N}(A; \rho_1, \rho_2, ...\rho_N) \left(\frac{g(C;A)}{\Delta} \right) \quad (11)$$

$$g(C;A) = \left(\frac{\vec{C}^S[\rho_1(C)]}{\vec{C}^S[\rho_1(A)]} \right) \left(\frac{\vec{C}^S[\rho_2(C)]}{\vec{C}^S[\rho_2(A)]} \right) \cdots \left(\frac{\vec{C}^S[\rho_N(C)]}{\vec{C}^S[\rho_N(A)]} \right)$$

Δ is the normalizing factor of summing the BBA to one.

3. Naïve Bayesian Fusion

Many real systems can be cast as having independent input probabilistic evidential data with an equal probable uniform prior probability distribution (i.e., Naïve Bayesian fusion). This greatly simplifies the mathematical techniques used to properly fuse the evidential data. In the past forty years [6] the Naïve Bayesian fusion has demonstrated many successes.

$$P(h_j \mid e_1, ..., e_n) = \frac{P(e_1 \mid h_j)...P(e_n \mid h_j)}{\sum_{h \in H} P(e_1 \mid h)...P(e_n \mid h)} \quad (12)$$

However, many studies [6,7] have also found that applying the Naive Bayesian classifier to the general conditional case to be surprisingly effective, despite the fact that its independence assumption is usually overly simplistic.

4. Dempster-Shafer (D-S) Belief Fusion

Combining two BBAs by using Dempster's rule of combination yields the fused BBA.

$$m_{1,2}^{DS}(A) = m_1 *_{DS}\ m_2(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - \sum_{B \cap C = \varnothing} m_1(B) m_2(C)} \quad (13)$$

The fused BBA is already normalized to sum to one.

$$\sum_{A \in 2^{\Omega}} m_1 *_{DS}\ m_2(A) = 1 \quad (14)$$

The same two BBAs can also be combined via the generalized belief fusion algorithm [13] having the probability proportionality weighting functions $\rho_J = \{1\}$ with Δ as the normalizing factor. The computed BBAs are the same as the D-S fused BBAs.

$$m_{1,2}^{DS}(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{1}{\Delta} \right) = m_{1,2}(A;1,1) \quad (15)$$

Calculating Δ as the normalizing factor of summing the fused BBA to one

$$\Delta = 1 - \sum_{B \cap C = \varnothing} m_1(B) m_2(C) \quad (16)$$

5. Equivalence Between Dempster-Shafer and Naïve Bayesian Fusion

This section derives the equivalence between Dempster-Shafer (D-S) and Naïve Bayesian Fusion is demonstrated. Initially the pignistic probability proportional to the normalized plausibility is calculated for the D-S fused BBAs. This result is shown to be equivalent to the Naïve Bayesian Fusion of the two PrNPl of the original BBAs.

Let the singleton $\{D, E\} \subseteq \Omega$ and the members of the power set $\{A, B, C\} \in 2^{\Omega}$, compute the pignistic probability proportional to the normalized plausibility (PrNPl) for the D-S fused BBAs.

$$\mathrm{PrNPl}_{1,2}(D) = \sum_{D \cap A \neq 0} m_{1,2}^{DS}(A) \left(\frac{1}{\Delta} \right) \quad (17)$$

Substituting the D-S fused BBAs

$$\mathrm{PrNPl}_{1,2}(D) = \sum_{D \cap A \neq 0} \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{1}{\Delta} \right) \quad (18)$$

Substituting for the A summation index

$$\mathrm{PrNPl}_{1,2}(D) = \sum_{D \cap B \cap C \neq 0} m_1(B) m_2(C) \left(\frac{1}{\Delta} \right) \quad (19)$$

Since D is a singleton:

$$\mathrm{PrNPl}_{1,2}(D) = \sum_{B \cap D \neq 0} m_1(B) \sum_{C \cap D \neq 0} m_2(C) \left(\frac{1}{\Delta} \right) \quad (20)$$

$$\mathrm{PrNPl}_{1,2}(D) = \sum_{B \cap D \neq 0} m_1(B) \sum_{C \cap D \neq 0} m_2(C) \left(\frac{1}{\Delta} \right) \quad (21)$$

$$\mathrm{PrNPl}_{1,2}(D) = \left(\frac{\sum_{B \cap D \neq 0} m_1(B) \sum_{C \cap D \neq 0} m_2(C)}{\Delta} \right) \quad (22)$$

Using the definition of PrNPl:

$$\mathrm{PrNPl}_{1,2}(D) = \left(\frac{\mathrm{PrNPl}_1(D)\ \mathrm{PrNPl}_2(D)}{\Delta} \right) \quad (23)$$

Substituting the normalization constant:

$$\mathrm{PrNPl}_{1,2}(D) = \left(\frac{\mathrm{PrNPl}_1(D)\ \mathrm{PrNPl}_2(D)}{\sum_{E \subseteq \Omega} \mathrm{PrNPl}_1(E)\ \mathrm{PrNPl}_2(E)} \right) \quad (24)$$

which is the form for the Naïve Bayesian fusion for the Pignistic Probability PrNPl.

6. Equivalence Between the Fixsen-Mahler Modified Dempster-Shafer and Naïve Bayesian Fusion

In this section the derivation of the equivalence between Modified Dempster-Shafer (MDS) and Naïve Bayesian Fusion is demonstrated. Initially Smets pignistic probability is calculated for the MDS fused BBAs. This result is shown to be equivalent the Naïve Bayesian Fusion of the two Smets pignistic probabilities of the original BBAs.

In literature, Fixsen-Mahler and Fister-Mitchell address the combination of two belief subsystems with a specific cardinality weighting called the Modified Dempster-Shafer Fusion (MDS):

The MDS fusion:

$$m_{1,2}^{MDS}(A) = \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{|A|}{\Delta |B||C|} \right) \quad (25)$$

Let the singleton $\{D, E\} \subseteq \Omega$ and the members of the power set $\{A, B, C\} \in 2^{\Omega}$, compute Smets pignistic probability for the MDS fused BBAs.

$$\mathrm{B}etP_{1,2}(D) = \sum_{D \subseteq A} m_{1,2}^{MDS}(A) \frac{1}{|A|} = \sum_{D \cap A \neq 0} m_{1,2}^{MDS}(A) \frac{1}{|A|} \quad (26)$$

Substituting the MDS fused BBAs:

$$\mathrm{B}etP_{1,2}(D) = \sum_{D \cap A \neq 0} \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{|A|}{\Delta |B||C|} \right) \frac{1}{|A|} \quad (27)$$

Cancel the cardinality of A.

$$\mathrm{B}etP_{1,2}(D) = \sum_{D \cap A \neq 0} \sum_{B \cap C = A} m_1(B) m_2(C) \left(\frac{1}{\Delta |B||C|} \right) \quad (28)$$

Substituting for the A summation index:

$$\mathrm{B}etP_{1,2}(D) = \sum_{D \cap B \cap C \neq 0} \frac{m_1(B) m_2(C)}{\Delta |B||C|} \quad (29)$$

Since D is a singleton:

$$\mathrm{Bet}P_{1,2}(D) = \sum_{D\cap B\neq 0} \frac{m_1(B)}{|B|} \sum_{D\cap C\neq 0} \frac{m_2(C)}{|C|} \frac{1}{\Delta} \quad (30)$$

Using the definition of Smets Pignistic Probability BetP:

$$\mathrm{Bet}P_{1,2}(D) = \left(\frac{\mathrm{Bet}P_1(D)\mathrm{Bet}P_2(D)}{\Delta} \right) \quad (31)$$

Substituting the normalization constant:

$$\mathrm{Bet}P_{1,2}(D) = \left(\frac{\mathrm{Bet}P_1(D)\mathrm{Bet}P_2(D)}{\sum_{E\subseteq\Omega} \mathrm{Bet}P_1(E)\mathrm{Bet}P_2(E)} \right) \quad (32)$$

which is the form for the Naïve Bayesian fusion for Smets Pignistic Probability BetP.

7. Equivalence Between the Sudano Generalized Belief Fusion Algorithm and Naïve Bayesian Fusion

In this section the derivation of the equivalence between the Generalized Belief Fusion Algorithm (GBFA) and Naïve Bayesian Fusion is demonstrated. Initially the pedigree pignistic probability is calculated for the GBFA fused BBAs. This result is shown to be equivalent the Naïve Bayesian Fusion of the corresponding pignistic probability of the original BBAs.

The Generalized Belief Fusion Algorithm of combining independent multi-source beliefs [10,13] is used to fuse N independent component sets $\{\mathrm{m}_1, \mathrm{m}_2, ..., \mathrm{m}_\mathrm{N}\}$ of basic belief assignments (BBAs).

$$m_{1,2,...,N}(A) = \sum_{B\cap C\cap ...\cap Z=A} m_1(B)m_2(C)...m_N(Z) \left(\frac{f(A;B,C,...,Z)}{\Delta} \right)$$

with

$$f(A;B,C,...,Z) = \left(\frac{\vec{C}^S[\rho_1(A)]}{\vec{C}^S[\rho_1(B)]} \right) \left(\frac{\vec{C}^S[\rho_2(A)]}{\vec{C}^S[\rho_2(C)]} \right) ... \left(\frac{\vec{C}^S[\rho_N(A)]}{\vec{C}^S[\rho_N(Z)]} \right) \quad (33)$$

$$m_{1,2,...,N}(A) = \sum_{B\cap C\cap ...\cap Z=A} \left(\frac{m_1(B)m_2(C)...m_N(Z))}{\Delta} \right) \left(\frac{\vec{C}^S[\rho_1(A)]}{\vec{C}^S[\rho_1(B)]} \right) \left(\frac{\vec{C}^S[\rho_2(A)]}{\vec{C}^S[\rho_2(C)]} \right) ... \left(\frac{\vec{C}^S[\rho_N(A)]}{\vec{C}^S[\rho_N(Z)]} \right) \quad (34)$$

Δ is the normalizing factor chosen so that the BBA sum to one.

For the singleton $C' \subseteq \Omega$, calculate the Pedigree Pignistic Probability (PrPed) of the GBFA fused BBAs.

$$\mathrm{PrPed}_{1,2,..N}(C') = \sum_{C'\subseteq A} m_{1,2,...,N}(A;\rho_1,\rho_2,...\rho_N) \left(\frac{g(C';A)}{\Delta} \right)$$

$$g(C';A) = \left(\frac{\vec{C}^S[\rho_1(C')]}{\vec{C}^S[\rho_1(A)]} \right) \left(\frac{\vec{C}^S[\rho_2(C')]}{\vec{C}^S[\rho_2(A)]} \right) ... \left(\frac{\vec{C}^S[\rho_N(C')]}{\vec{C}^S[\rho_N(A)]} \right) \quad (35)$$

Δ is the normalizing factor of summing the BBAs to one

$$\mathrm{PrPed}_{1,2,..N}(C') = \sum_{C\subseteq A} \frac{m_{1,2,...,N}(A;\rho_1,\rho_2,...\rho_N)}{\Delta} \left(\frac{\vec{C}^S[\rho_1(C')]}{\vec{C}^S[\rho_1(A)]} \right) \left(\frac{\vec{C}^S[\rho_2(C')]}{\vec{C}^S[\rho_2(A)]} \right) ... \left(\frac{\vec{C}^S[\rho_N(C')]}{\vec{C}^S[\rho_N(A)]} \right) \quad (36)$$

Substituting the fused GBFA BBA into the Pedigree Pignistic equation and canceling terms

$$\mathrm{PrPed}_{1,2,..N}(C') = \sum_{C\subseteq A} \sum_{B\cap C\cap ...\cap Z=A} \left(\frac{m_1(B)m_2(C)...m_N(Z))}{\Delta} \right) \left(\frac{1}{\vec{C}^S[\rho_1(B)]} \right) \left(\frac{1}{\vec{C}^S[\rho_2(C)]} \right) ... \left(\frac{1}{\vec{C}^S[\rho_N(Z)]} \right) \left(\frac{\vec{C}^S[\rho_1(C')]}{1} \right) \left(\frac{\vec{C}^S[\rho_2(C')]}{1} \right) ... \left(\frac{\vec{C}^S[\rho_N(C')]}{1} \right) \quad (37)$$

$$\mathrm{PrPed}_{1,2,..N}(C') = \sum_{C'\subseteq A} \sum_{B\cap C\cap ...\cap Z=A} \left(\frac{m_1(B)m_2(C)...m_N(Z))}{\Delta} \right) \left(\frac{\vec{C}^S[\rho_1(C')]}{\vec{C}^S[\rho_1(B)]} \right) \left(\frac{\vec{C}^S[\rho_2(C')]}{\vec{C}^S[\rho_2(C)]} \right) ... \left(\frac{\vec{C}^S[\rho_N(C')]}{\vec{C}^S[\rho_N(Z)]} \right) \quad (38)$$

$$\mathrm{PrPed}_{1,2,..N}(C') = \sum_{C'\subseteq A} \sum_{B\cap C\cap ...\cap Z=A} \left(\frac{1}{\Delta} \right) \left(\frac{m_1(B)\vec{C}^S[\rho_1(C')]}{\vec{C}^S[\rho_1(B)]} \right) \left(\frac{m_2(C)\vec{C}^S[\rho_2(C')]}{\vec{C}^S[\rho_2(C)]} \right) ... \left(\frac{m_N(Z)\vec{C}^S[\rho_N(C')]}{\vec{C}^S[\rho_N(Z)]} \right) \quad (39)$$

Since $C' \subseteq \Omega$ is a singleton, this allows a simplification of the summation indices.

$$\{C' \subseteq A \;,\; B\cap C\cap ...\cap Z = A\},$$
$$\{B\cap C\cap ...\cap Z = A \supseteq C'\},$$
$$\begin{Bmatrix} B\cap A = A\supseteq C' \\ C\cap A = A\supseteq C' \\ ... \\ Z\cap A = A\supseteq C' \end{Bmatrix}, \quad \begin{Bmatrix} B\supseteq C' \\ C\supseteq C' \\ ... \\ Z\supseteq C' \end{Bmatrix} \quad (40)$$

$$\mathrm{PrPed}_{1,2,..N}(C') = \left(\frac{1}{\Delta} \right) \sum_{C'\subseteq B} \left(\frac{m_1(B)\vec{C}^S[\rho_1(C')]}{\vec{C}^S[\rho_1(B)]} \right) \sum_{C'\subseteq C} \left(\frac{m_2(C)\vec{C}^S[\rho_2(C')]}{\vec{C}^S[\rho_2(C)]} \right) ... \sum_{C'\subseteq Z} \left(\frac{m_N(Z)\vec{C}^S[\rho_N(C')]}{\vec{C}^S[\rho_N(Z)]} \right) \quad (41)$$

$$\mathrm{PrPed}_{1,2,..\mathrm{N}}(C') = \frac{\mathrm{PrPed}_1(C')\mathrm{PrPed}_2(C')...\mathrm{PrPed}_\mathrm{N}(C')}{\sum_{D'\subseteq\Omega} \mathrm{PrPed}_1(D')\mathrm{PrPed}_2(D')...\mathrm{PrPed}_\mathrm{N}(D')} \quad (42)$$

which is the form for the Naïve Bayesian fusion for the Pedigree Pignistic Probabilities.

This can further be simplified since C' is a singleton, $\{C'\} \subseteq \Omega$ allows the simplification $\vec{C}^S[\rho_1(C')] = \rho_1(C')$

$$\mathrm{PrPed}_{1,2,..N}(C') = \left(\frac{1}{\Delta} \right) \sum_{C'\subseteq B} \left(\frac{m_1(B)\rho_1(C')}{\vec{C}^S[\rho_1(B)]} \right) \sum_{C'\subseteq C} \left(\frac{m_2(C)\rho_2(C')}{\vec{C}^S[\rho_2(C)]} \right) ... \sum_{C'\subseteq Z} \left(\frac{m_N(Z)\rho_N(C')}{\vec{C}^S[\rho_N(Z)]} \right) \quad (43)$$

using the definitions of the pignistic probability in ( )

$$\mathrm{PrPed}_{1,2,..N}(C') = \left(\frac{1}{\Delta} \right) \Pi_1(C')\Pi_2(C')...\Pi_N(C') \quad (44)$$

Substituting the normalization constant:

$$\mathrm{PrPed}_{1,2,\dots N}(C') = \left(\frac{\Pi_1(C')\Pi_2(C')\dots\Pi_N(C')}{\sum_{D' \subset \Omega} \Pi_1(D')\Pi_2(D')\dots\Pi_N(D')} \right) \qquad (45)$$

which is the form for the Naïve Bayesian fusion for the appropriate Pignistic Probability.

8. Conclusion

The process of fusing multiple independent sensor measurements, communication link data from other independent systems, and dynamic data base information is essential to support critical decisions in a timely way. Many real systems can be mapped to such a process. The independence of the input evidential data with an equal probable uniform prior probability distribution (i.e., Naïve Bayesian fusion) greatly simplifies the mathematical techniques used to properly fuse the evidential data. Equivalence between the Pignistic Probability Estimates of the Belief Fusion of the BBAs and the Naïve Bayesian fusion of the Pignistic Probability Estimates of the individual BBAs has been shown for this process. The equivalence comparison is done in probability space.

The practical implications of this paper are quite interesting for the information fusion process in many real systems. For many such systems, some inputs to the information fusion process are better represented by the exponential belief, $\text{Power-set}(\Omega) = 2^{\Omega}$, representation of the incomplete information set. Via an appropriate pignistic probability transform, all these inputs are mapped into the linear probability Ω set representations and fused. This greatly simplifies the computation complexity since the equivalent fusion results are obtained in linear probability space rather than exponential belief space.

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