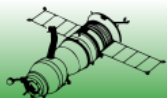


Uncertainty Quantification (UQ) in Complex System of Systems (SoS) Modeling and Simulation (M&S) Environments

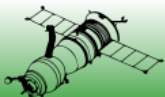
Paper #65

Deiotte, Garrett, Marvin, Morantz,
Whalen



Agenda

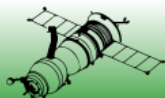
- Motivation
- Uncertainty Quantification (UQ) Approach
- SoS Observations and Future Work



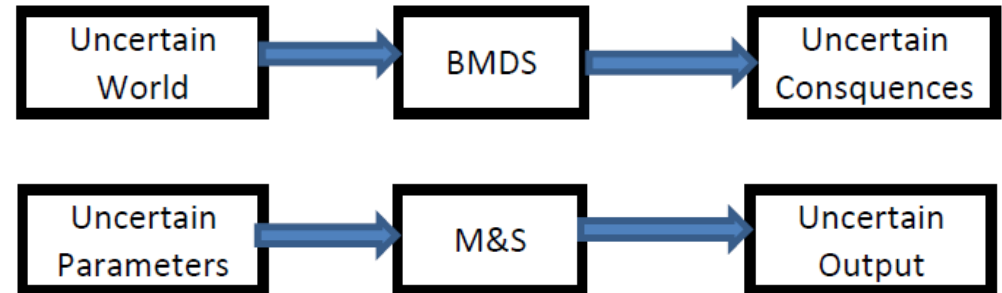
Motivation

- UQ research inspired by a Missile Defense Agency Scientific Technology Transfer (STTR) topic

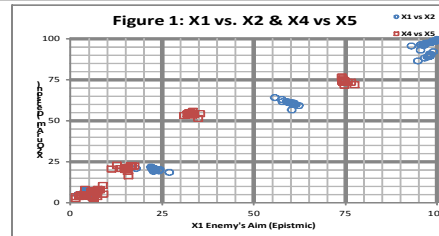
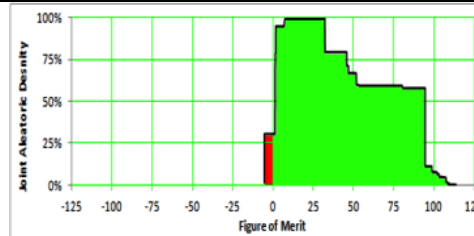
Develop and demonstrate Uncertainty Quantification (UQ) capabilities for Ballistic Missile Defense System (BMDS) Modeling and Simulation (M&S). Include methods and tools for efficiently, effectively specifying, representing and analyzing both epistemic (known unknown) and aleatoric (unknown unknown) uncertainties affecting BMDS outcomes.



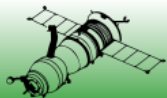
Uncertainty Quantification (UQ)

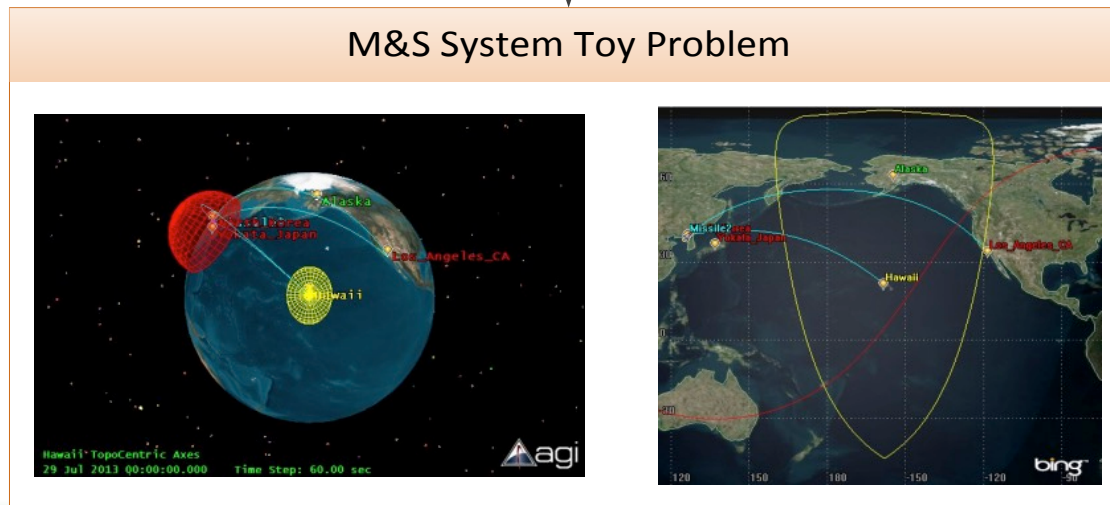
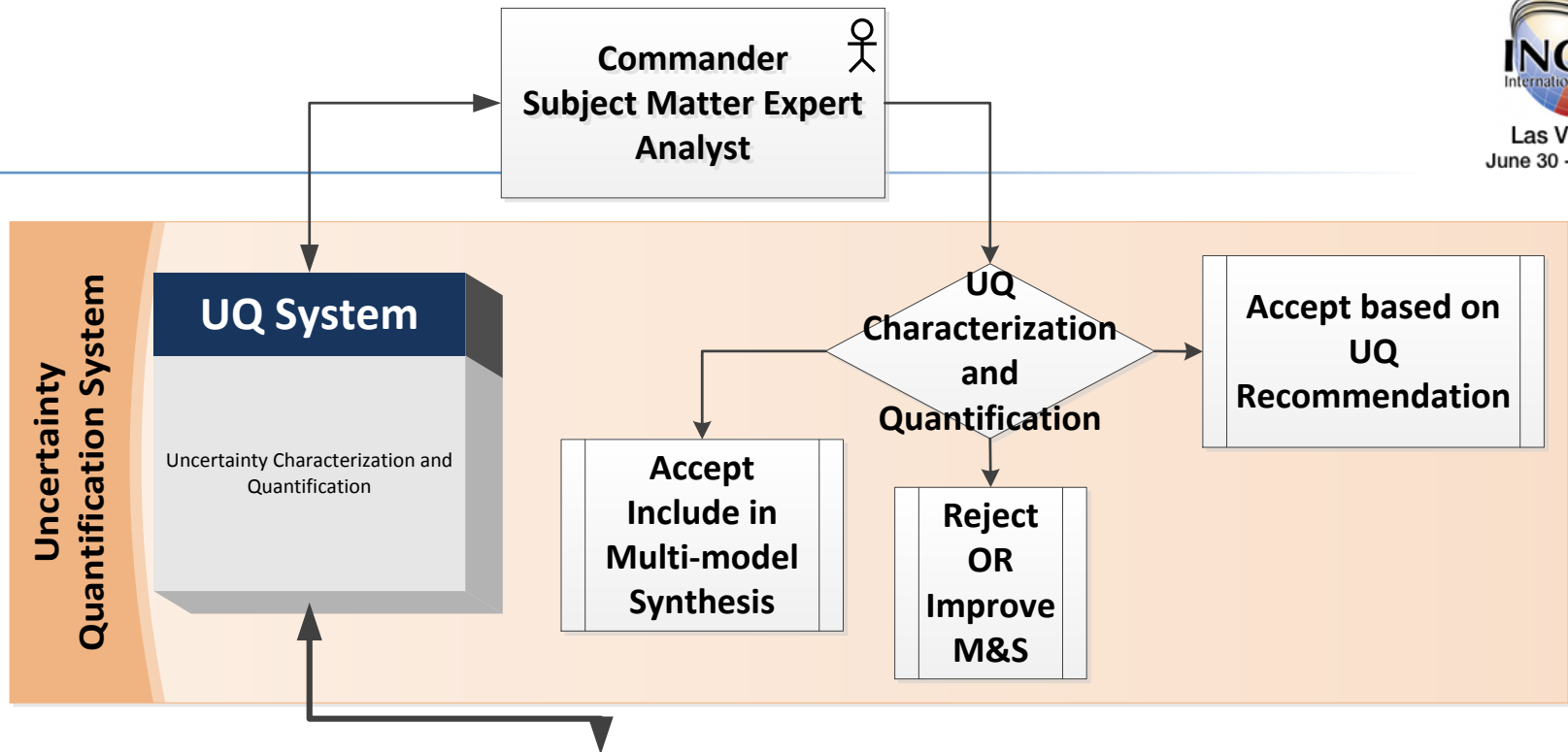


Aleatoric
Uncertainty

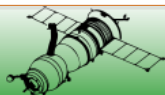


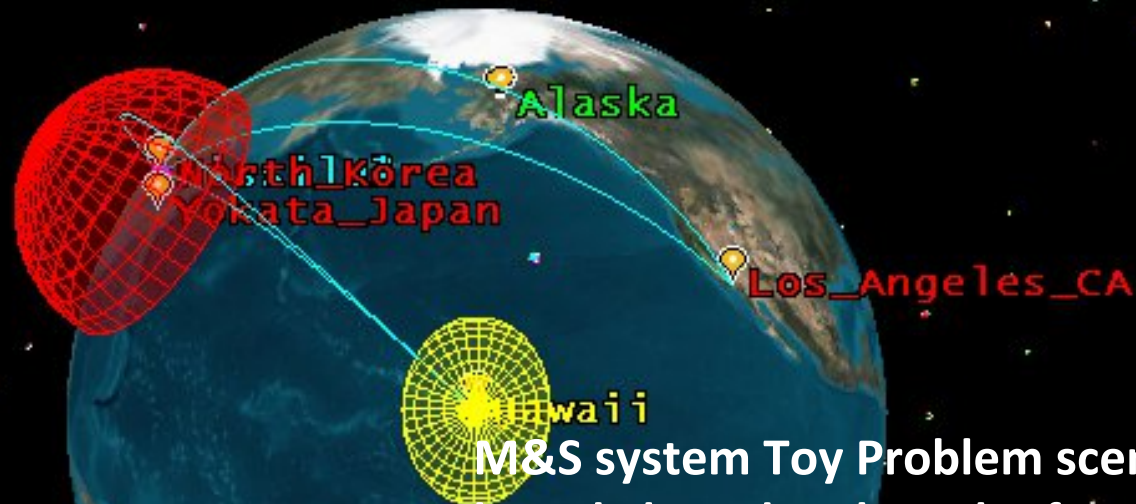
Global
Uncertainty
Local
Uncertainty





UQ System acts on M&S Systems (Toy Problem) to characterize and quantify uncertainty

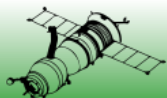




M&S system Toy Problem scenario is loosely based on launch of a N. Korean UNHA-3 missile . Launch envelop ranges from Hawaii to west coast of U.S (Los Angeles). Sensors at Yokota, Japan and Hawaii acquire and track the target. Intercept missiles in Alaska and Aegis (blue dot).

Hawaii TopoCentric Axes
29 Jul 2013 00:00:00.000

Time Step: 60.00 sec

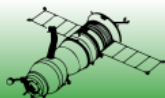


Parameters in Toy Problem

Parameters:

- x1 First enemy aim point, no probability attached
- x2 First friendly aim point, no probability attached
- x3 First kill radius, normally distributed
- x4 Second enemy aim point, no probability attached
- x5 Second friendly aim point, no probability attached
- x6 Second kill radius, normally distributed
- These 6 parameters create a 6 dimensional parameter space in our Toy Problem implemented in spreadsheet

Parameters are uncertain inputs to M&S Systems

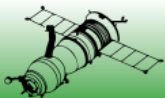


We are in a SoS - Now What?



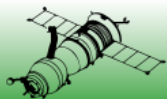
Join INCOSE SoSWG!

- Pain Point: Autonomy, Interdependencies and Emergence
 - What are the analysis, prediction and architecture tools needed?
- Think about our Toy Problem in a SoS context
- Develop a UQ concept



Biologically Inspired UQ Approach

- A non-intrusive Uncertainty Quantification and Characterization System that performs statistical and parametric analyses to characterize the uncertainty and allow:
 - Selection of the best M&S System and correct or improve the rest
 - Perform a M&S system multi-model synthesis
- UQ System encompasses uncertainty quantification, subject matter experts and resulting decisions

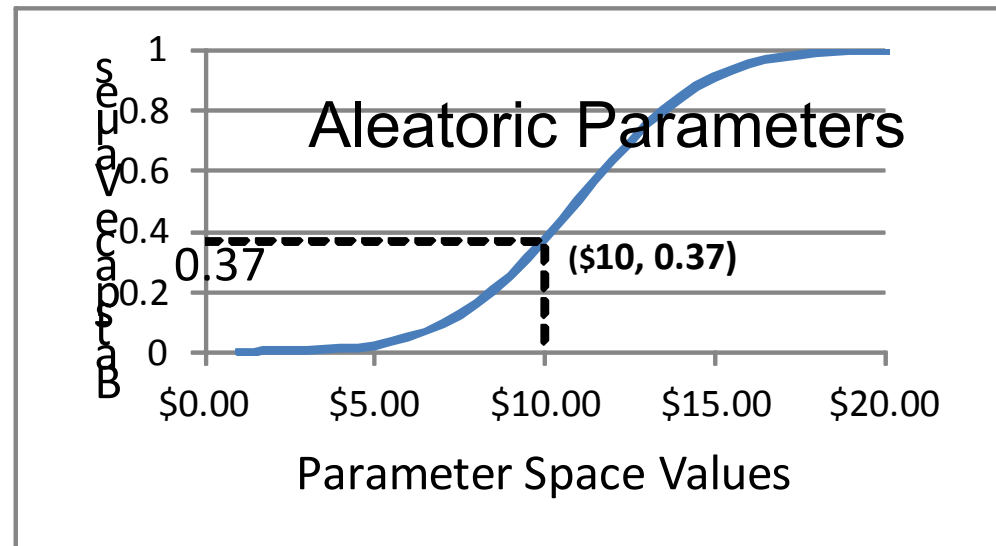


5 Step UQ Algorithm

- Step One: Consistent Treatment of Aleatoric and Epistemic Parameters.

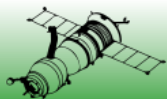
Epistemic
Parameters

+



Probit
Transformation –
Don't throw away evidence
Based prob. info

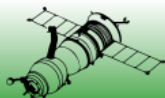
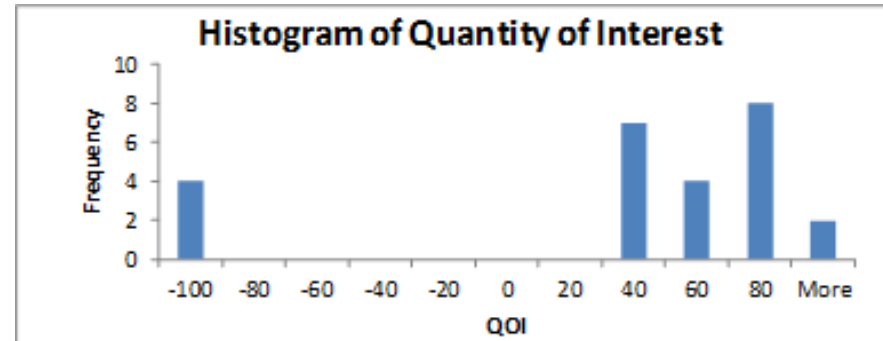
➔ Multidimensional “Batspace”



5 Step UQ Algorithm

- Step Two: Standard Statistical Characterization and Quantification of Uncertainty using the Latin Hypercube sampling approach to generate random inputs to exercise the M&S under evaluation.

Mean	40.1
Standard Deviation	59.9
Mode	-100.0
Kurtosis	1.9
Skewness	-1.7
Minimum	-100.0
25th Percentile	30.3
Median	59.5
75th Percentile	75.1
Maximum	91.8
Range	191.8
Interquartile Range	44.7
Sum	1203.2
Count	30.0
Estimated Probability of Failure	13.3%



5 Step UQ Algorithm

- Step Three: Search for Areas of Noteworthy Performance
 - Exploited search of “batspace” data in (Step One) unit Hypercube



Belfry 5 Final Hard TW July 15.xls [Compatibility Mode] - Microsoft Excel

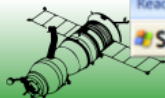
Home Insert Page Layout Formulas Data Review View Developer Acrobat

G11 $=Page1!$J$1+Bspace!G11*(Page1!$K$1-Page1!$J$1)$

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
1	Pspace!: First six rows in each block of seven = input variables X1, X2,... in natural units for one batch of what-if analyses																			
2	Seventh row in each block of seven = Figure of Merit in natural units																			
3	Time	Bat #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
4	1	X1	14.6	24.7	3.445	7.159	36.06	36.58	13.57	29.13	34.52	21.73	46.95	12.71	85.4	75.3	96.55	92.84	63.94	63.42
5		X2	86.56	19.94	47.89	31.52	61.56	75.11	13.44	80.06	52.11	68.48	38.44	24.89	71.87	56.12	41.25	33.74	81.78	11.75
6		X3	120.3	99.99	95.16	109.1	112.5	116.1	99.71	120	124.8	110.9	107.5	103.9	120.1	119.1	128.2	117	125.9	112.9
7		X4	30.89	4.512	12	2.187	45.35	10.67	69.11	95.49	88	97.81	54.65	10.29	34.37	39.44	32.16	38.11	45.49	89.71
8		X5	26.32	34.27	75.12	34.42	19.06	70.61	73.68	65.73	24.88	65.58	80.94	29.39	76.59	53.81	52.48	74.63	54.13	12.66
9		X6	112.9	109.5	113.5	110.9	116.1	99.7	107.1	110.5	106.5	109.1	103.9	120.3	144	129.8	117.9	106.1	107.2	116.4
10	1	f(X)	46.45	77.62	48.06	76.76	74.59	37.32	97.5	65.54	41.04	60.02	72.89	89.3	96.95	90.49	70.76	55.58	53.97	37.75
11	2	X1	20.51	17.32	10.96	14.92	26.83	29.46	17.96	31.49	30.62	26.2	44.43	16.74	79.97	73.15	89.83	87.46	64.47	64.16
12		X2	83.68	30.17	54.52	43.23	73.35	69.71	21.42	70.14	58.07	63.73	42.12	30.01	75.49	45.85	34.82	29.36	68.47	26
13		X3	119.1	98.64	102.2	111	115.7	112.2	105.1	122	118.7	114.5	110.8	107.2	121.8	117.7	122.5	116.3	122.1	114.9
14		X4	24.93	7.243	11.82	4.427	39.34	11.08	73.64	93.57	92.14	95.53	63.24	24.31	37.97	52.1	45.83	49.56	54.87	81.68
15		X5	38.74	49.27	74.85	44.22	22.49	72.33	64.52	50.71	38.65	52.72	68.23	28.88	70.77	44.31	43.77	62.09	46.3	21.15
16		X6	109.6	111	110.9	108.5	114.8	103.7	107.1	109.1	107.5	108.4	104.7	117.4	119.6	124.6	117.8	108.4	109.1	114.6
17	2	f(X)	53.51	66.36	45.49	66.43	64.88	40	92.01	63.74	51.45	63.23	78.72	90.8	79.11	71.97	64.84	55.54	55.92	51.45
18	3	X1	23.91	14.22	18.23	21.27	27.58	29.91	21.21	30.52	30.84	30.58	39.23	19.98	73.67	69.78	76.21	79.86	64.2	63.84
19		X2	78.44	43.05	60.68	54.75	72.27	69.36	31.7	58.67	60.29	58.98	47.72	36.1	72.59	38.01	40.01	28.76	55.34	25.92
20		X3	116.2	100.6	106.5	111.6	114.7	112.3	108.7	118.6	117	118.5	113.4	109.6	121.9	116.6	121.4	115.9	119.4	115
21		X4	19.91	9.326	11.88	7.252	32.16	10.81	78.34	92.06	74.83	92.21	73.88	38.57	44.48	63.77	56.57	59.37	63.34	82.02
22		X5	51.67	67.71	73.0	66.78	35.07	72.32	67.24	38.8	45.8	30.87	57.60	30.97	61.2	35.4	47.62	49.18	38.51	21.15
23		X6	112.9	109.5	113.5	110.9	116.1	99.7	107.1	110.5	106.5	109.1	103.9	120.3	144	129.8	117.9	106.1	107.2	116.4
24		f(X)	46.45	77.62	48.06	76.76	74.59	37.32	97.5	65.54	41.04	60.02	72.89	89.3	96.95	90.49	70.76	55.58	53.97	37.75

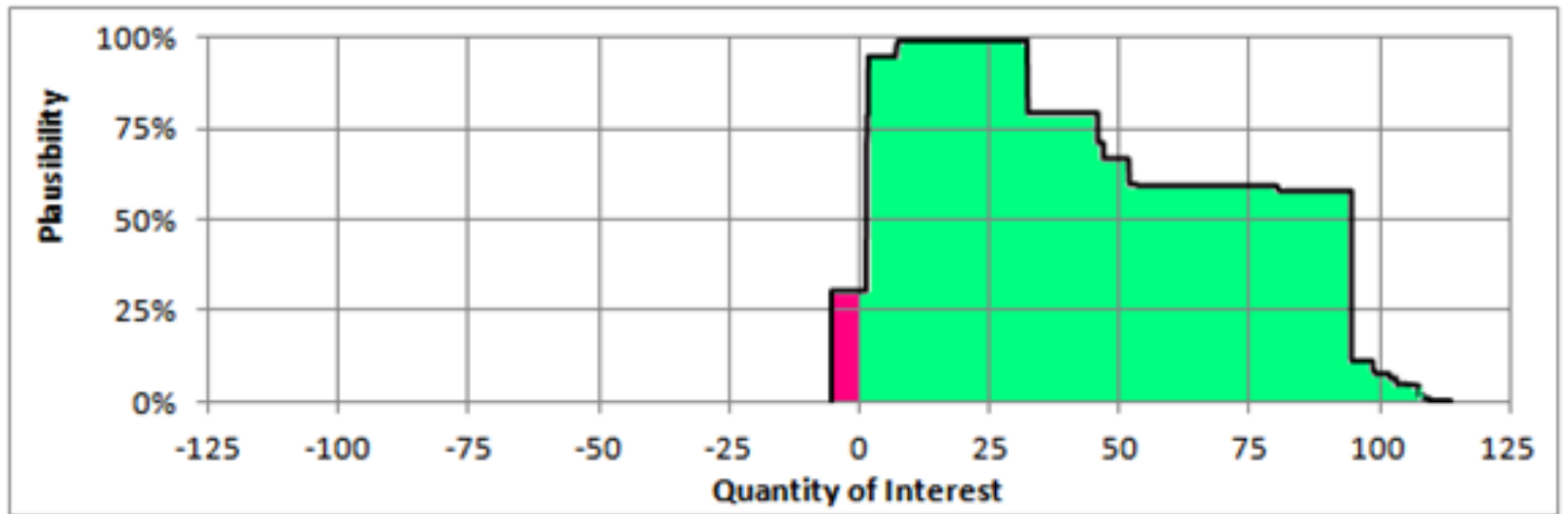
Ready Average: 65.42416743 Count: 6 Sum: 392.5450046 90%

Start Inbox for ... R: Arithm... Hard TW Slode... Microsoft... 4:35 PM

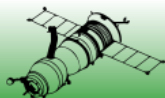


5 Step UQ Algorithm

- Step Four: Global Characterization and Quantification of Uncertainty

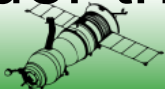


Evidence-Based Fuzzy Set of Plausible
Values of Margin



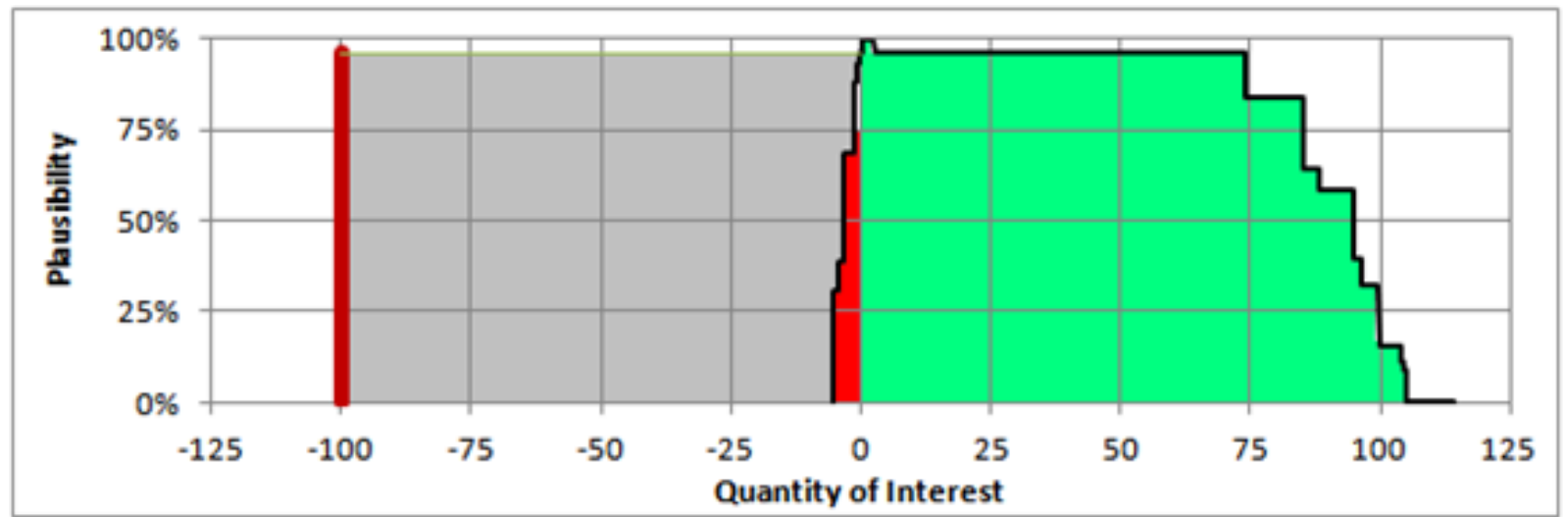
Fuzzy Set of Plausible Values of Margin (cont)

- Plausibility of Failure = maximum height of red area = 30.5%
- Plausibility of Success = maximum height of green area = 100%
- Certainty of Failure = $1 - \text{Plausibility of Success} = 0$
- Probability of Failure is between 0 and 30.5%
- the spread is due to the presence of Epistemic uncertainty in the input parameters
- Global Quantity of Uncertainty = Total area under the curve = 73.7 (same units as margin)

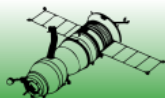


5 Step UQ Algorithm

- Step Four: Global Characterization and Quantification of Uncertainty



Area Under Curve = Quantitative Measure of Total combined
Statistical and Possibilistic QOI = UQ = 184.94



5 Step UQ Algorithm

- Step Five: Local Characterization and Quantification of Uncertainty

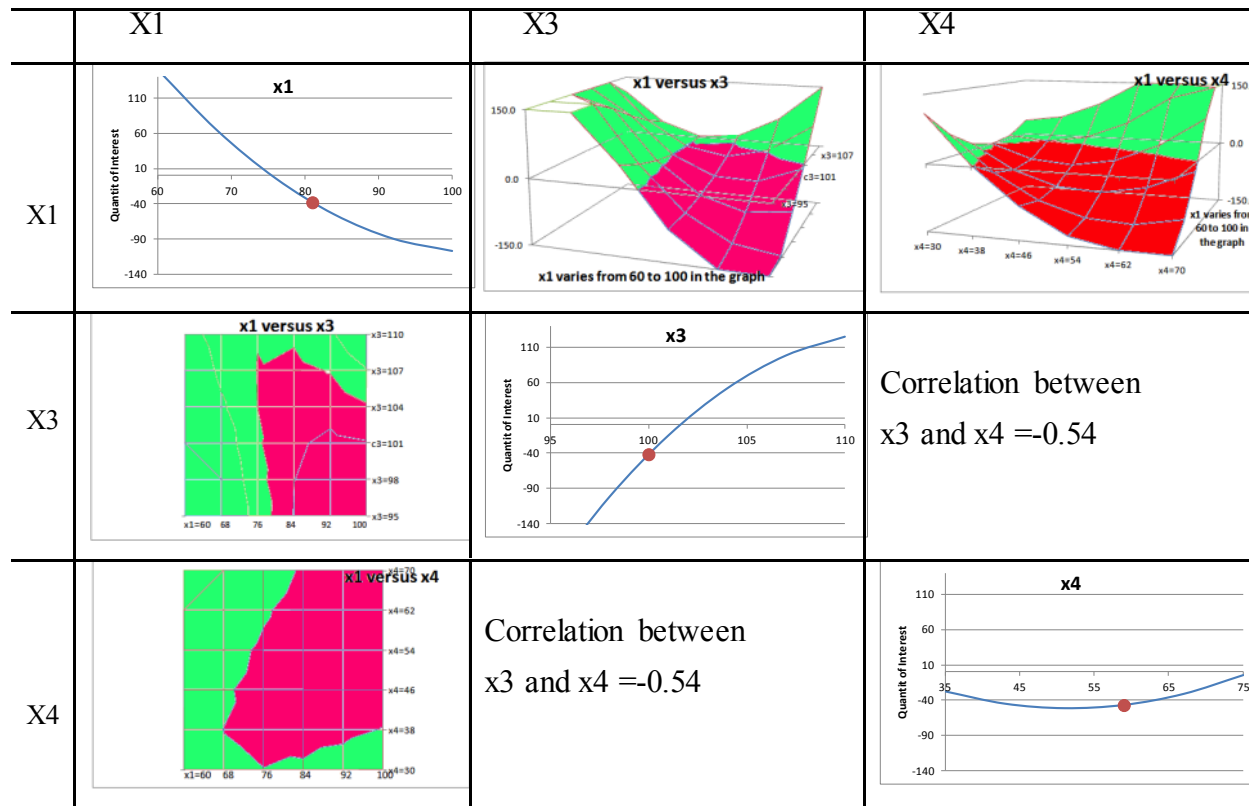
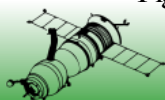
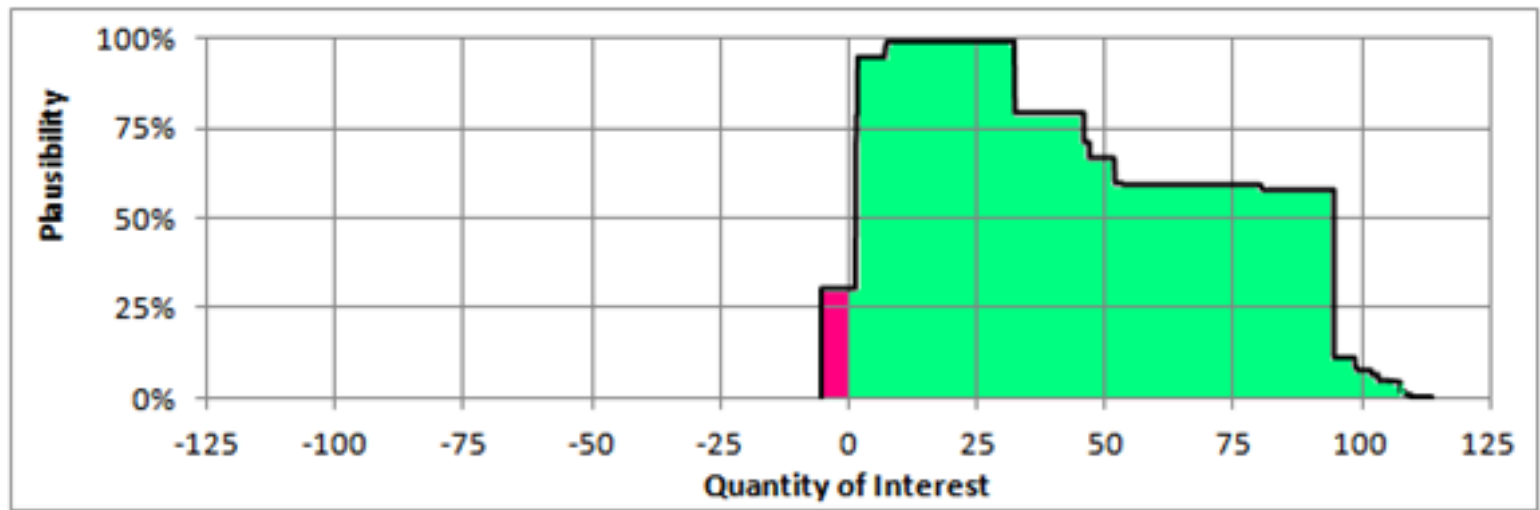


Fig. 8: Graphical Characterization and Quantification of Example ANP

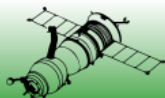


5 Step UQ Algorithm

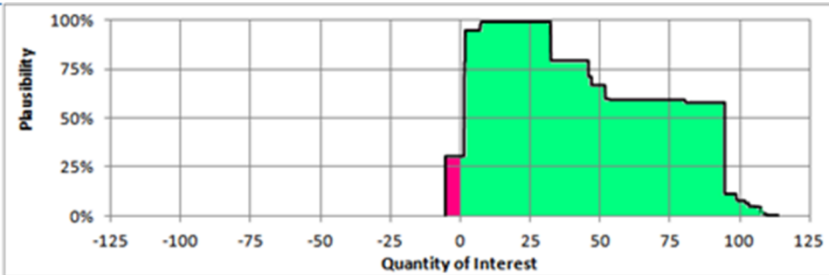
- Step Four: Global Characterization and Quantification of Uncertainty



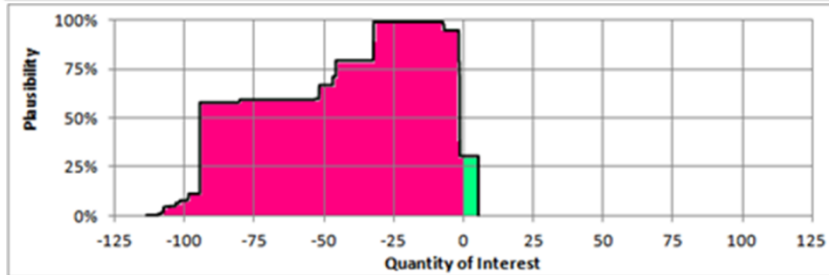
Evidence-Based Fuzzy Set of
Plausible Values of Margin



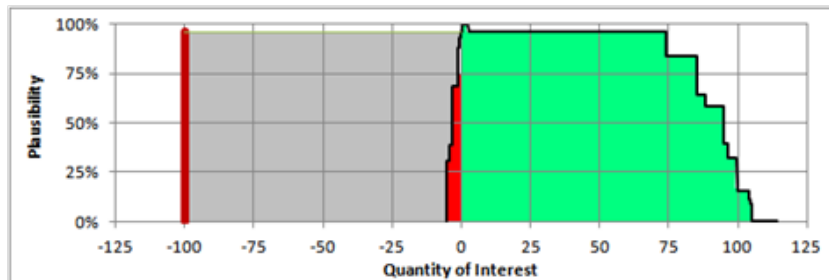
Example Results



System A: Optimistic
M&S System



System B: Pessimistic
M&S System



System C: Sensitive
M&S System

Plausibility vs Quantity of Interest (QOI) Legend

Characterization

Positive Outcome .

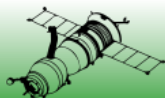
Characterization

Negative Outcome.

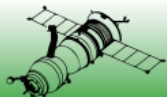
Characterization

No Observed Values.

Total Area under Curve = Uncertainty Quantity

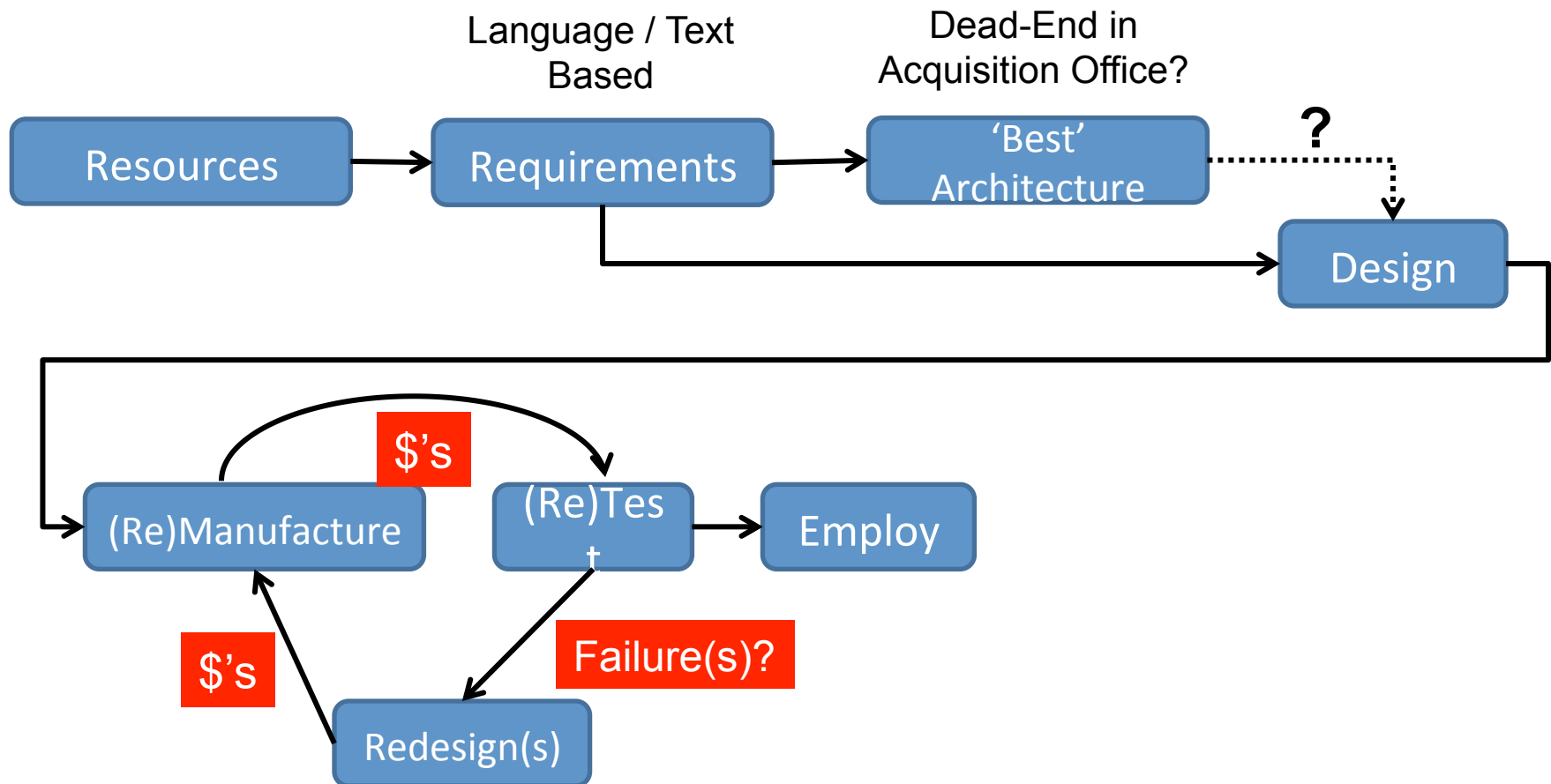


SoS Observations and Future Work



The Current state in Developing a SoS

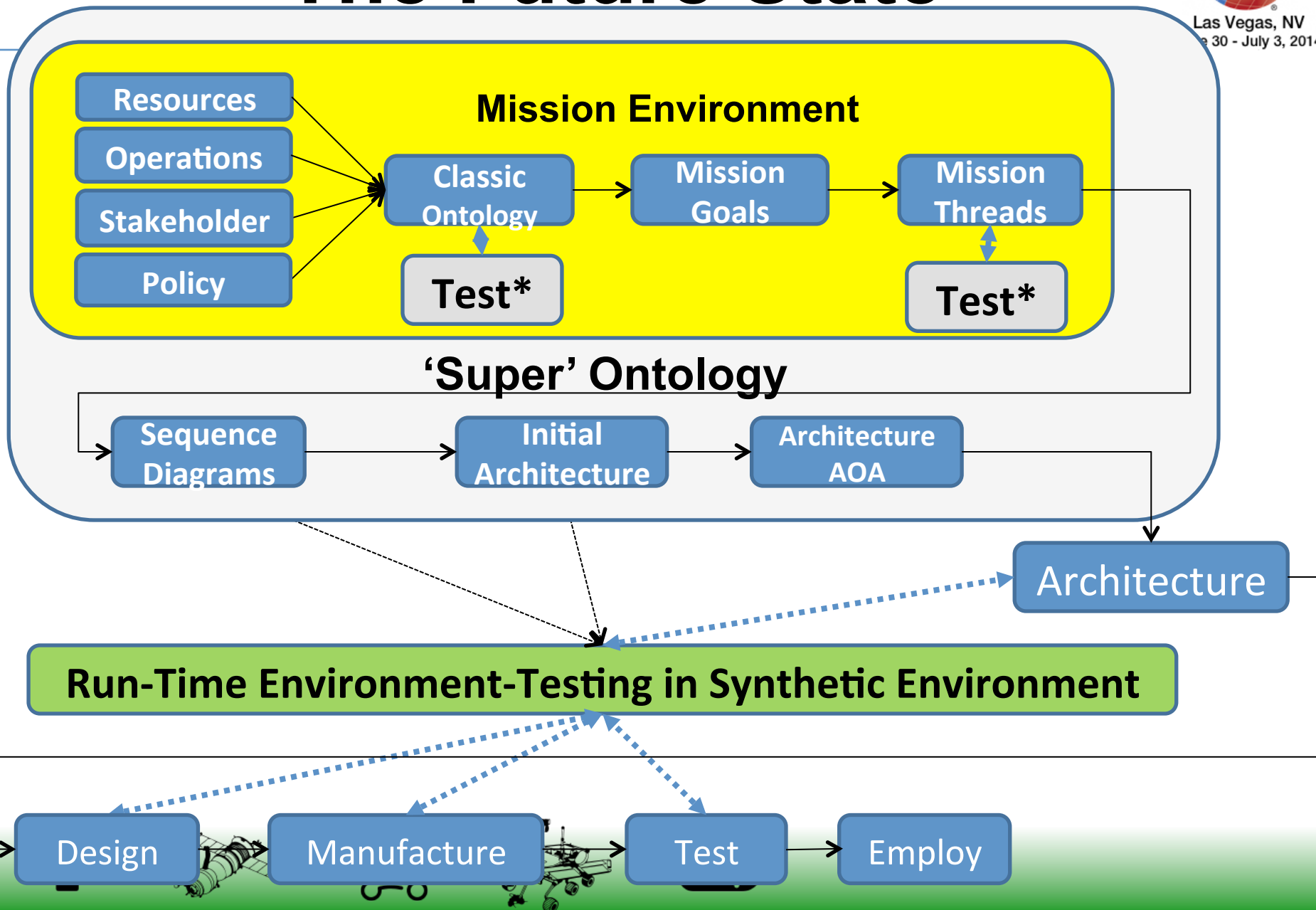
(or a Software Intensive, Complex System)



Architecture and Design are difficult to test until initial capability is manufactured

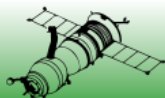


The Future State



Summary

- Toy SoS Problems become complex fast!
- Putting engineering back into systems engineering essential
- New tools and techniques coming into view that can motivate a shift in SoSE
 - Graph Theory (SysML)
 - Data Analytics
 - UQ and related tools



UCQ Commercialization Roadmap

