25th anniversary annual INCOSE international symposium
Seattle, WA
July 13 - 16, 2015

Fundamentals of System Complexity Measures for Systems Design

Dr Mahmoud Efatmaneshnik
&
Dr Mike Ryan

UNSW Australia Canberra

Capability Systems Centre
UNSW Canberra

Complexity measures

- A predominant preoccupation in systems engineering as well as in product engineering has been the development of justifiable complexity measures that can be of assistance to system designers, acquirers and operators.
- In particular, we are keen to identify complexity metrics for engineered systems that correlate with and predict project cost, schedule or reliability, which can also be used to compare designs /system concepts alternatives.
- However, there is no consensus about how to measure complexity in the context of systems engineering, which is further exacerbated by the fact that the applicability of such complexity measures to predict project success remains fundamentally dubious.

Complexity measures

- Here we are motivated by the observation that previous considerations of appropriate measures have not directly addressed the fundamental issue that the complexity of any particular matter or thing has a significant subjective component in which the degree of difficulty depends on available frames of reference.
- Any attempt to remove subjectivity from a measure of complexity therefore fails to address a very significant aspect of complexity.

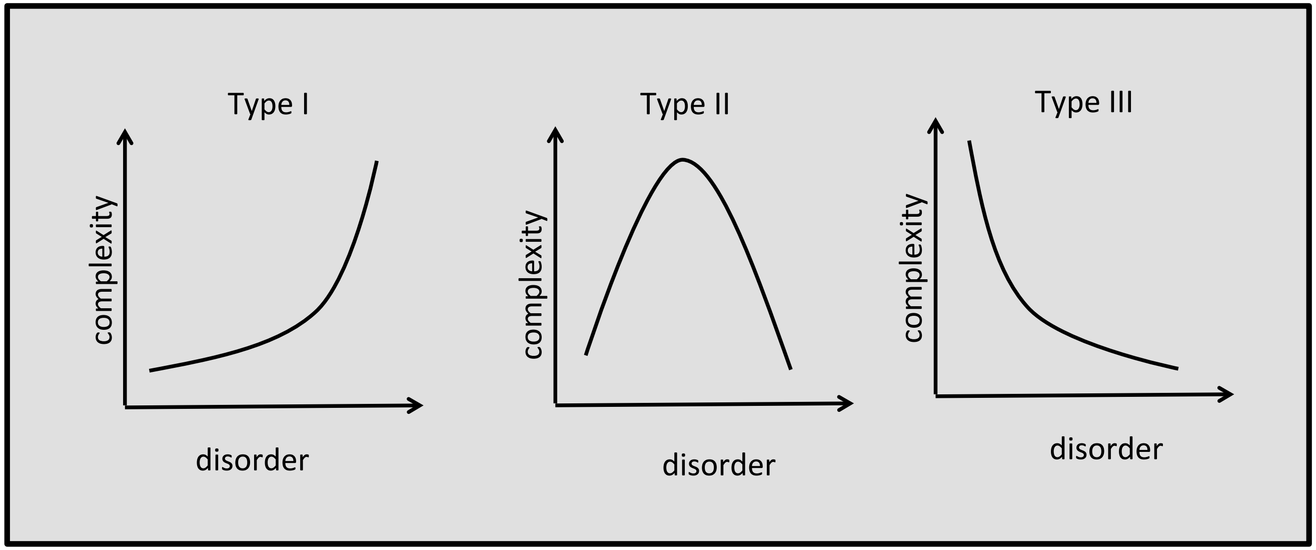
Complexity measures

- On the other hand, there has been justifiable apprehension towards purely subjective complexity measures, simply because they are not verifiable if the frame of reference being applied is in itself both complex and subjective.
- We address this issue by introducing the concept of subjective simplicity—although a justifiable and verifiable value of subjective complexity may be difficult to assign directly, it is possible to identify with high accuracy what is a ‘simple’ structure and, from that reference, determine subjective complexity.

Complexity measure

- Although there are numerous complexity measures in many fields, each tends to be context-dependent within that field.
- We propose a general framework for measures for complexity that accounts for previous measures, in particular for previous attempts at defining a universal complexity measure.

Types of complexity measure



- Type I complexity measure strictly increases with the amount of information regardless of whether this information is useful, has patterns, or is structured.
- Type II measures capture useful information only (as opposed to total information); capturing information content, structure, organization, patterns, regularities and symmetries in systems.
- Type III complexity measures relate to self-organizing systems.

Shiner, J.S., M. Davison, and P.T. Landsberg. 1999. "Simple measure for complexity." *Physical Review E* 59 (2):1459-1464.

The search for dual aspect of complexity

- In order to obtain a Type II measure, Shiner et al defined:

$$\Gamma_{\alpha,\beta} = \Delta^{\alpha}(1-\Delta)^{\beta}$$

$$\Delta = \frac{H[X]}{H_{max}[X]}$$

$$H_{max}[\mathrm{X}] = \log N$$

- where $H[X]$ is Boltzman-Gibbs-Shannon entropy and H_{max} is its maximum possible entropy. α, β are to be chosen depending on the context.

The search for dual aspect of complexity

- Feldman and Crutchfield proposed two statistical complexity measures by multiplying disorder and a distance function of the current state of the system from complete disorder:

$$C[Y] = H[Y]D[Y]$$

where *D*[*X*] is the distance from a uniform distribution.

- Note that the distance could be measured from any given known distribution.
- We extend this notion to be the distance from **reference simplicity**.

A general complexity measure

- We propose that a complexity measure comprises two components:

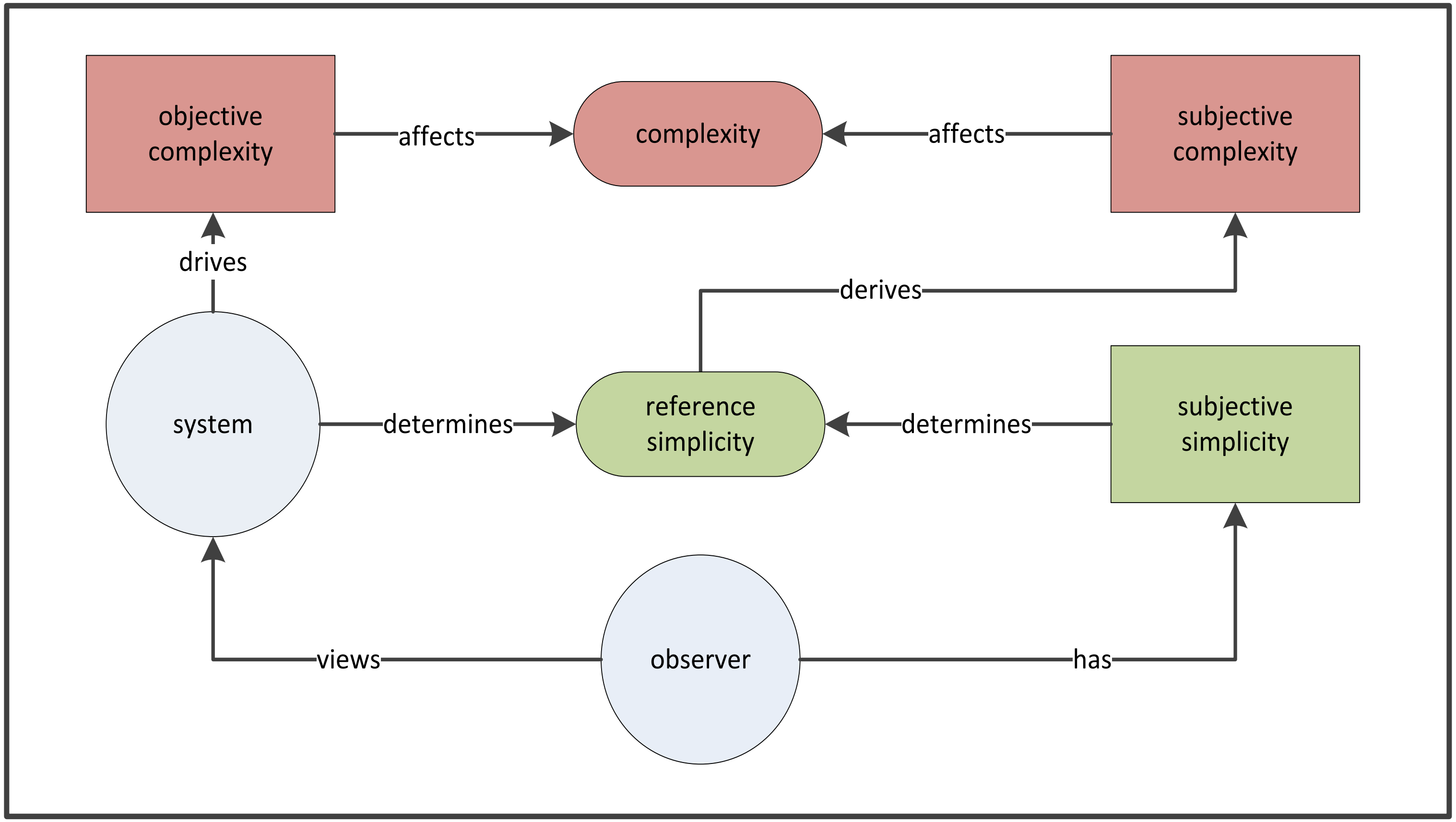
$$\mathcal{K} = |\mathcal{S}| \times \mathcal{D}(\mathcal{S} \| \mathcal{R})$$

- where $\mathcal{K}$ is complexity, |•| denotes objective complexity, $\mathcal{D}(\bullet)$ is a distance function, $\mathcal{S}$, is the object of study, and $\mathcal{R}$ is the reference model of simplicity for that object category from a given view point or a subjective context.

A general complexity measure

- That is, we propose that the two components are an objective component and a subjective component:
  - *Proposition I:* Objective complexity is a measure of the extent of a system's minimal description.
  - *Proposition II:* Subjective complexity is a measure of the departure from the observer's reference simplicity.

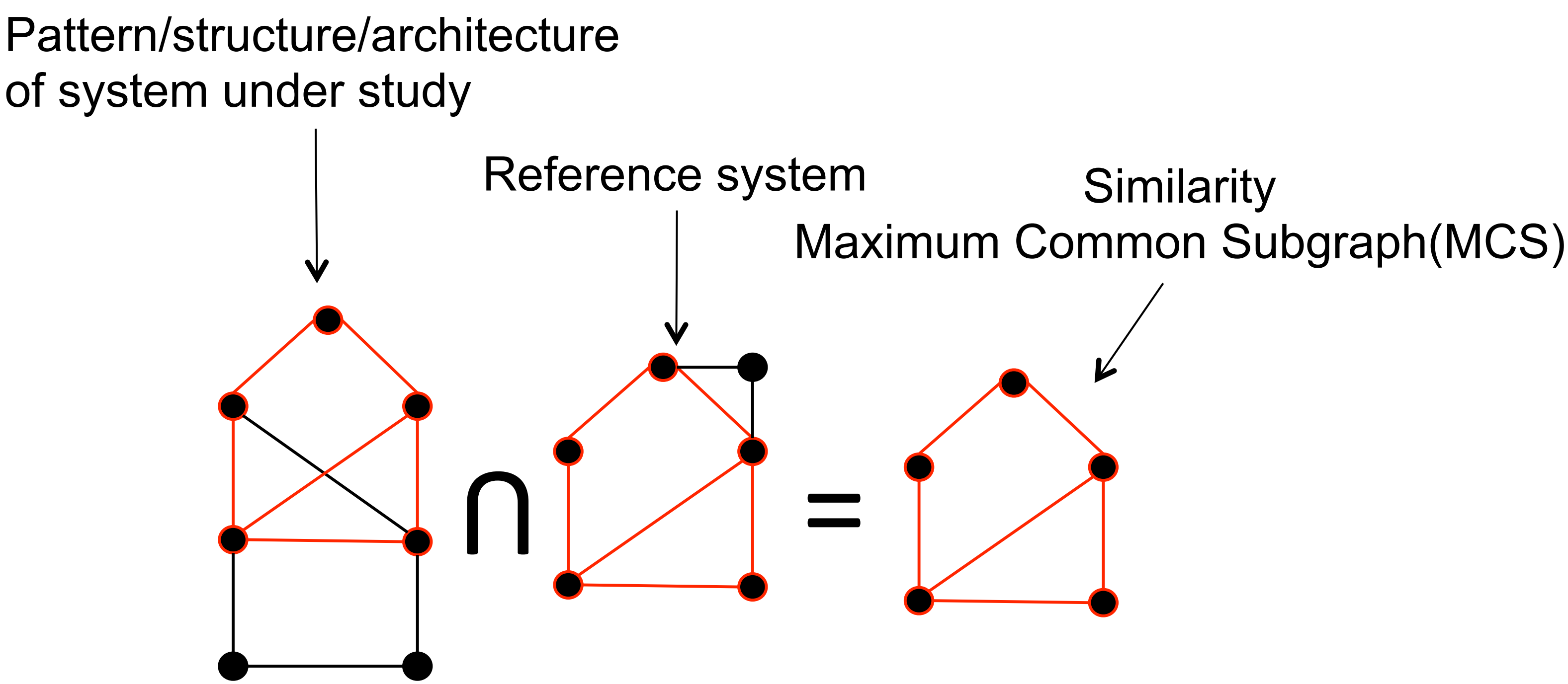
A general complexity measure



Objective system complexity

- If a system is “a combination of interacting elements organized to achieve one or more stated purposes”, it can be physically decomposed hierarchically into a directed graph.
- An objective measure of system complexity must therefore be a graph (theoretic) measure, or a colored graph for including system variety.
  - Numbers of nodes / links / nodes + links / nodes x links
  - Number of subgraphs
  - Graph energy
  - Maximum eigenvalue

Subjective system complexity



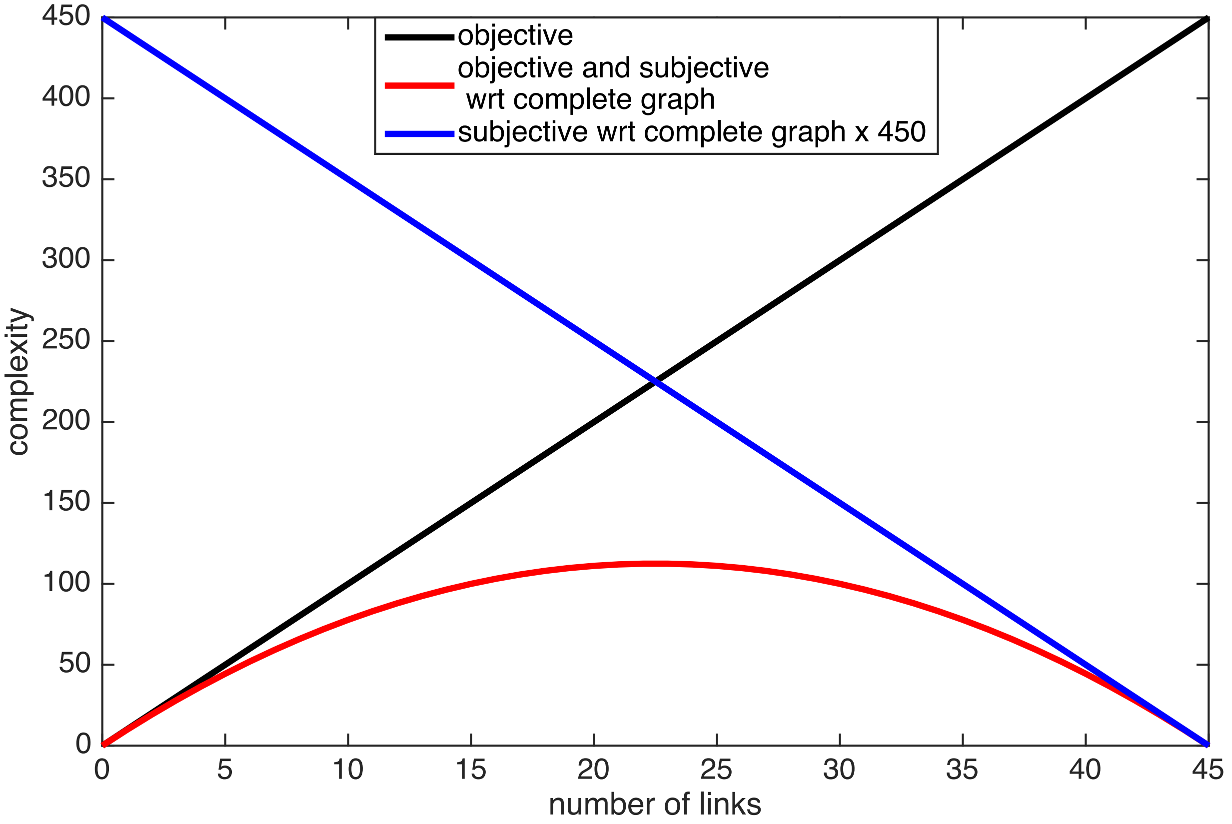
Distance of system from reference:

$$D(G,R) = 1 - \frac{|MCS(G,R)|}{|G| + |R| - |MCS(G,R)|}$$

|•| denoted a size measure e.g. number of nodes etc.

Simple example

- Assume objective complexity is (nodes x links) and reference simplicity is a complete (fully connected) graph.
- Shows the complexity of a graph of maximum 10 nodes with a varied number of links (any randomly generated graph).



Example - Cyclomatic complexity

- Cyclomatic complexity is the number of independent loops in a graph. For a connected graph *G* with *m* links and *n* nodes, cyclomatic complexity:

$$\text{Cyc}(G) = m - n + 1$$

- If we assume a spanning tree (*ST*) of graph *G* as its reference and the objective complexity measure as:

$$|G| = n + m$$

$$\text{MCS}(G,ST) = ST\text{, and } |ST| = 2n - 1$$

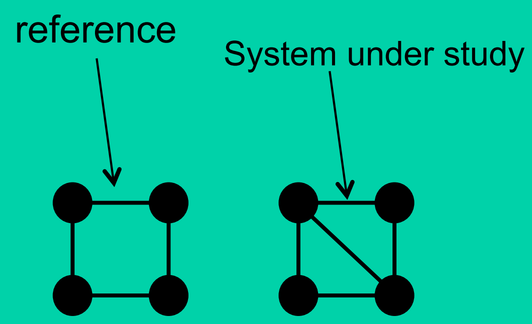
- Then it is trivial that to see that cyclomatic complexity is generated by:

$$\text{Complexity }(G) = |G| \times \text{D}(G \,||\, ST) = \text{Cyc}(G)$$

$$\text{where D}(G||ST) = 1 - |ST| \,/\, |G|$$

Therefore, in effect, cyclomatic complexity expresses a subjective desire for tree-like structures, and is achieved by supposing reference simplicity as spanning tree, and also assuming (number of nodes + number of links) as objective complexity

Example 3



$$C_{sys\_objetive} = 20$$

$$C_{reference\_objective} = C_{mcs\_objective} = 16$$

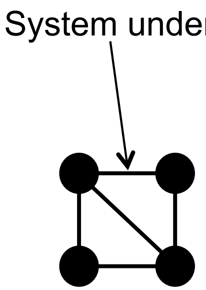
$$C_{sys\_subjetive} = 0.2$$

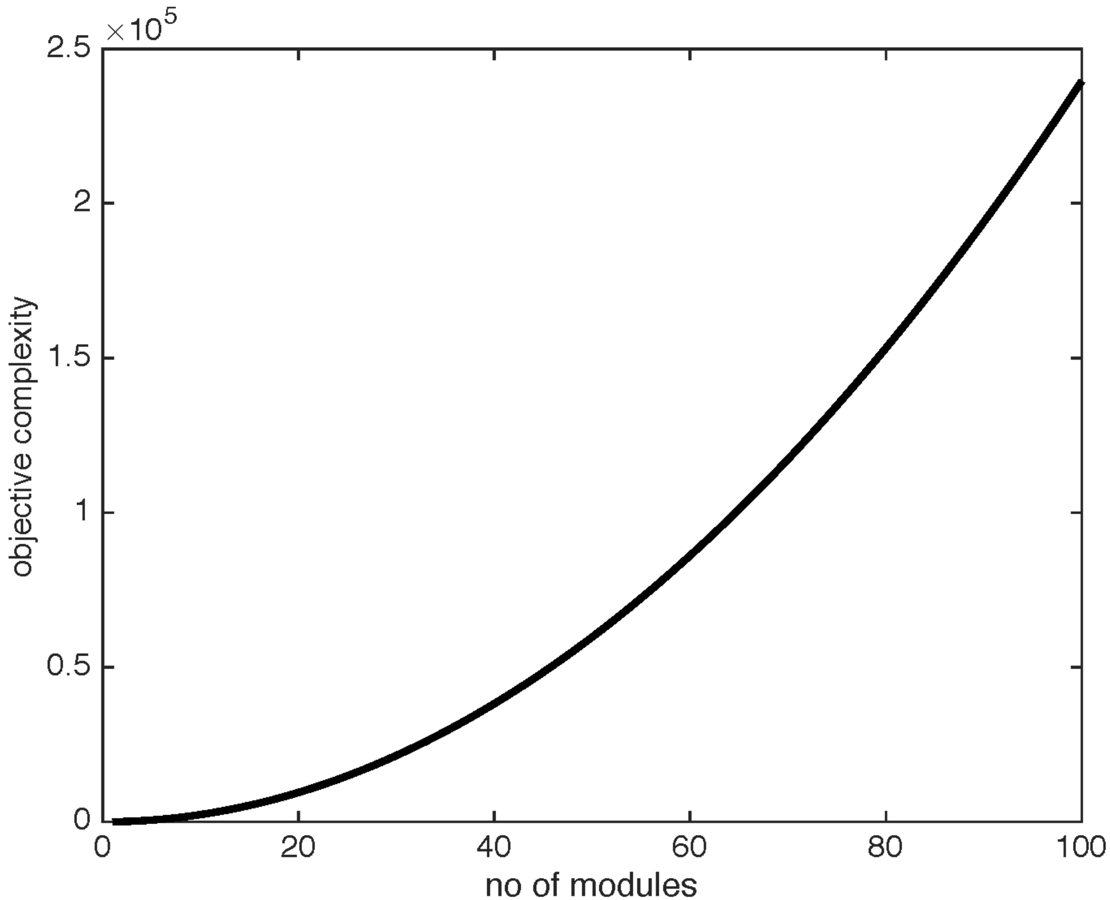
$$C_{sys} = C_{sys\_objetive} \times C_{sys\_subjetive} = 4$$

Assume $|G| = \text{nodes} \times \text{links}$

$$D(G,R) = 1 - \frac{|MCS(G,R)|}{|G| + |R| - |MCS(G,R)|}$$

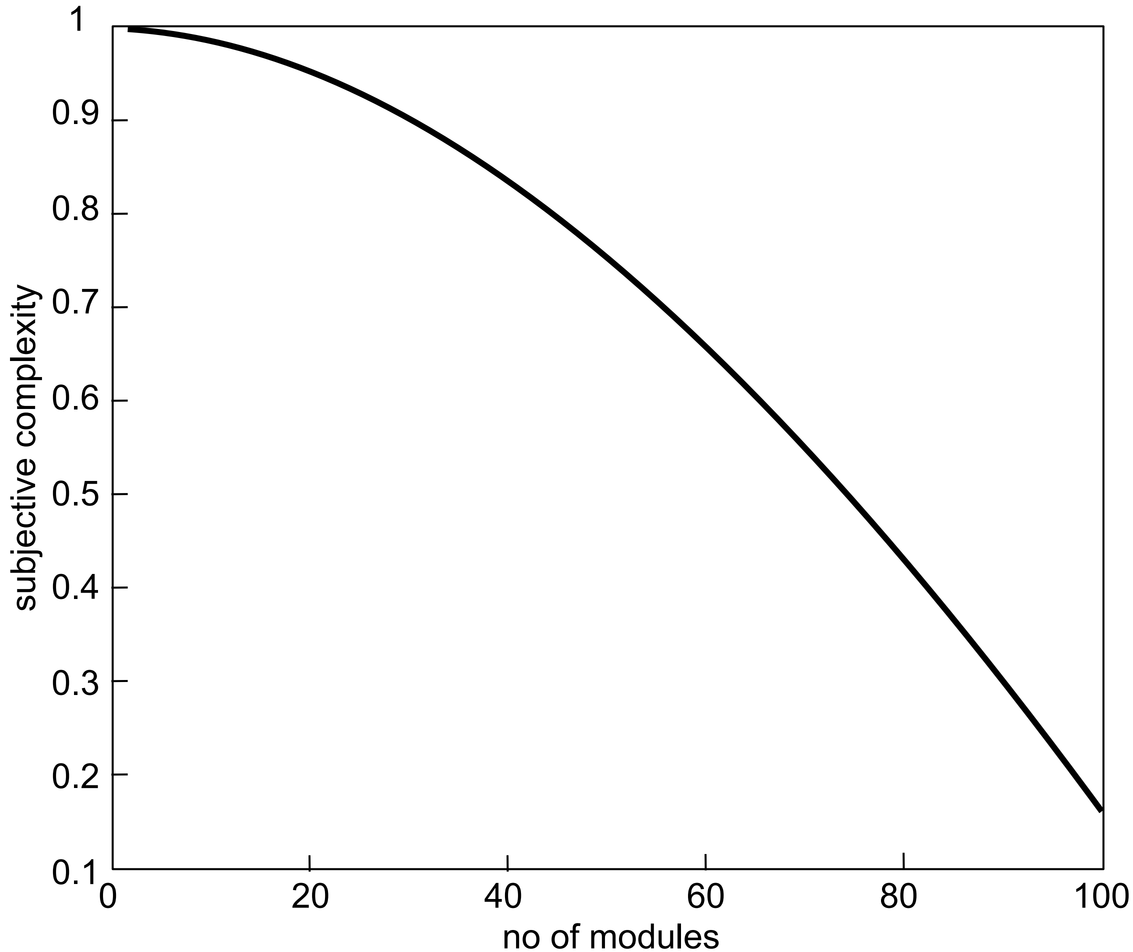
Objective system complexity





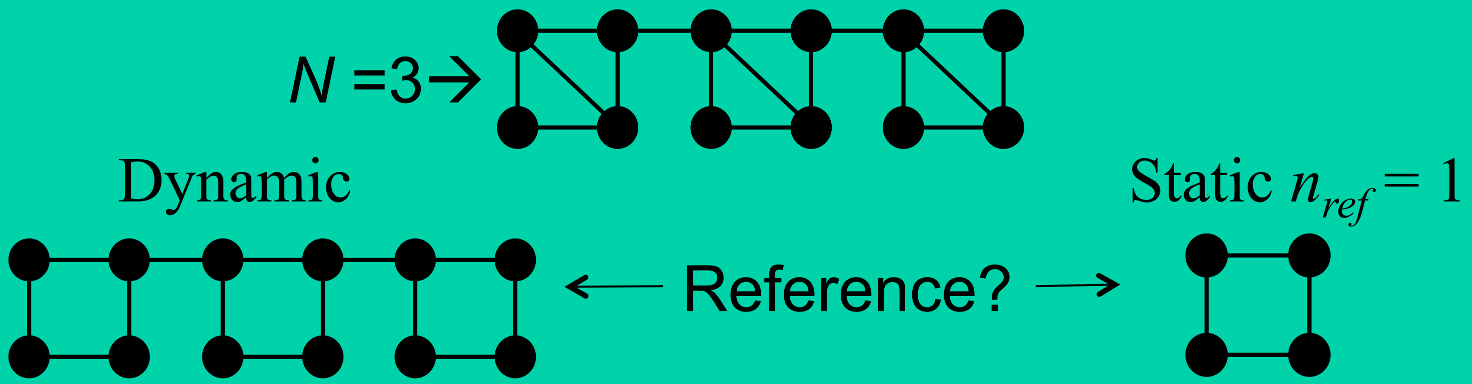
With reference to a fully connected graph of 100 nodes

Subjective system complexity



With reference to a fully connected graph of 100 nodes

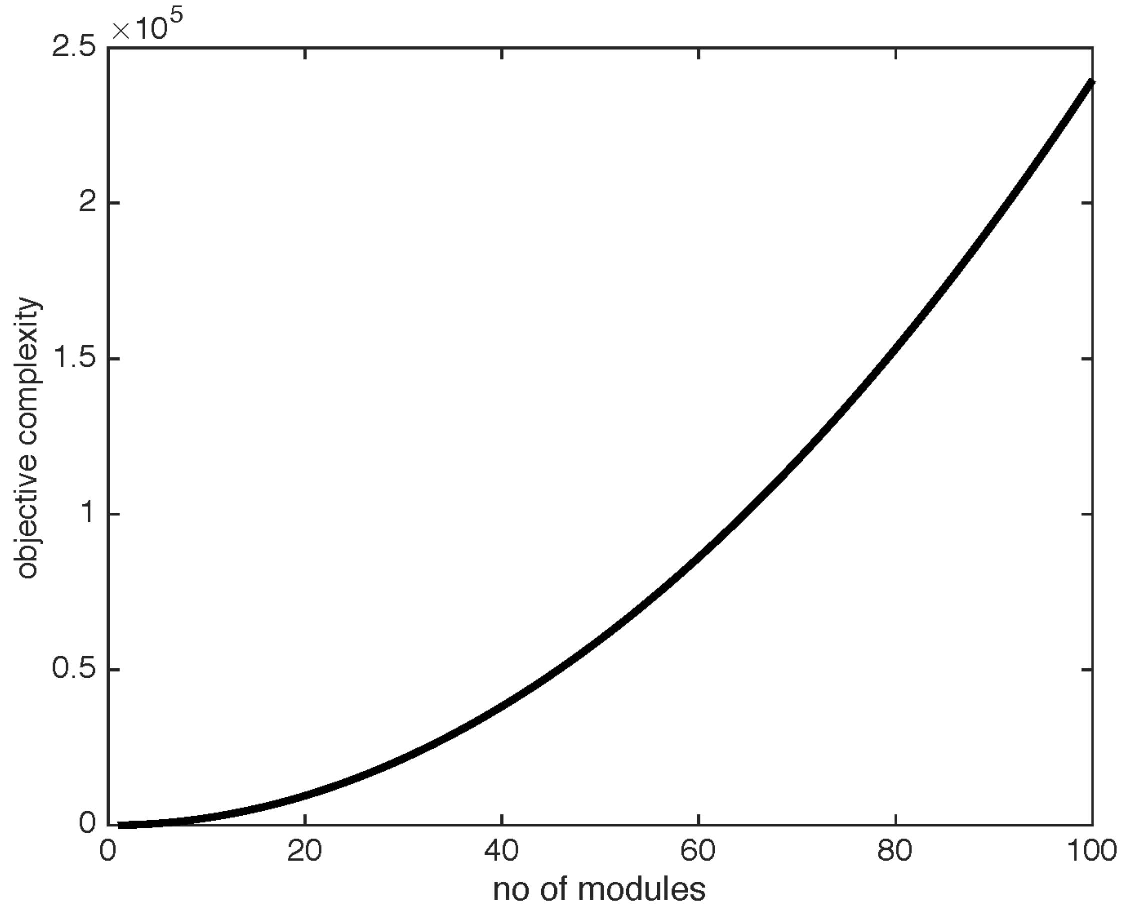
Example 4



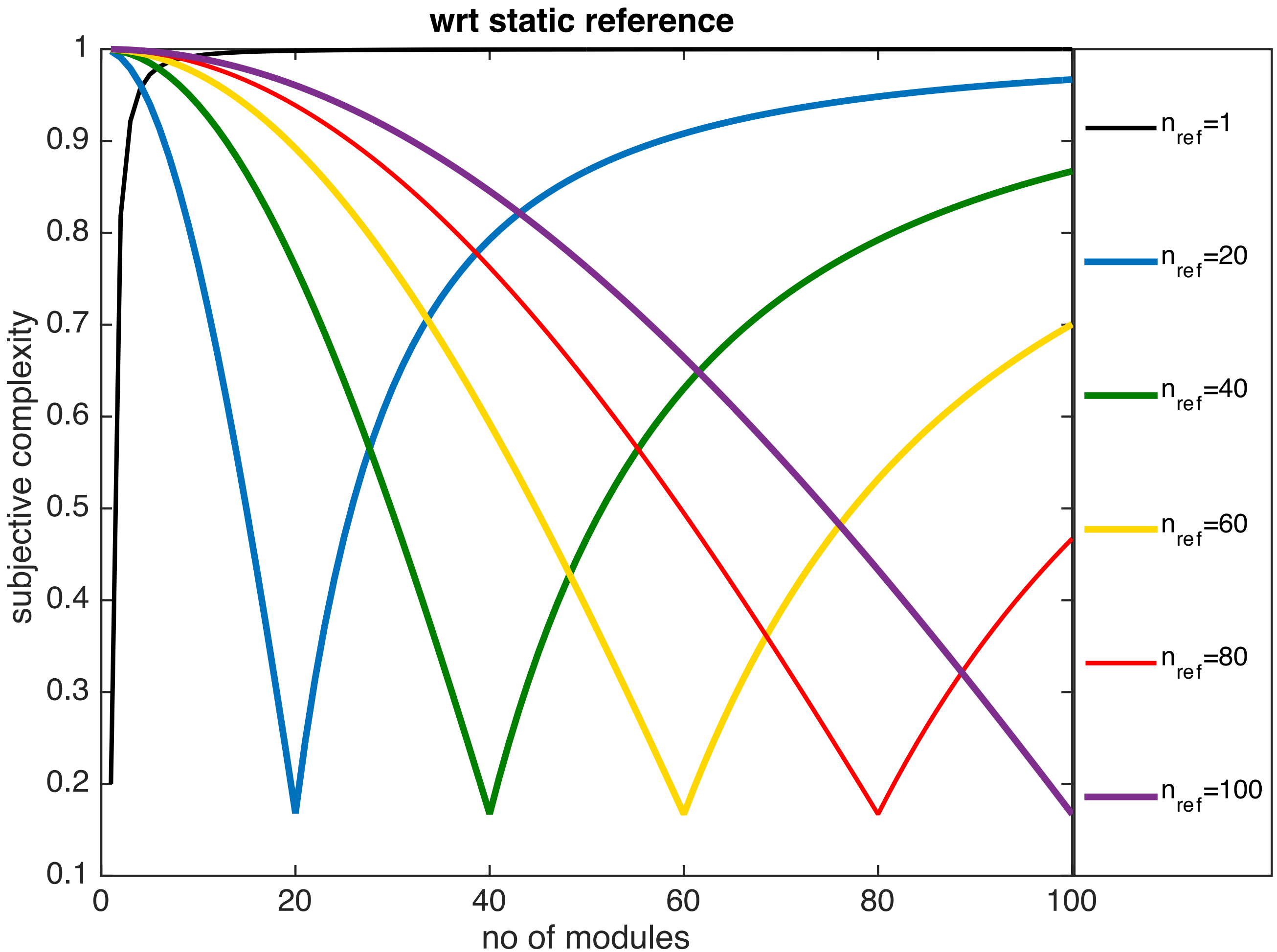
Dynamic reference model grows with system size (left) up to a maximum.

Static reference does not (right) change as the size of the system changes.

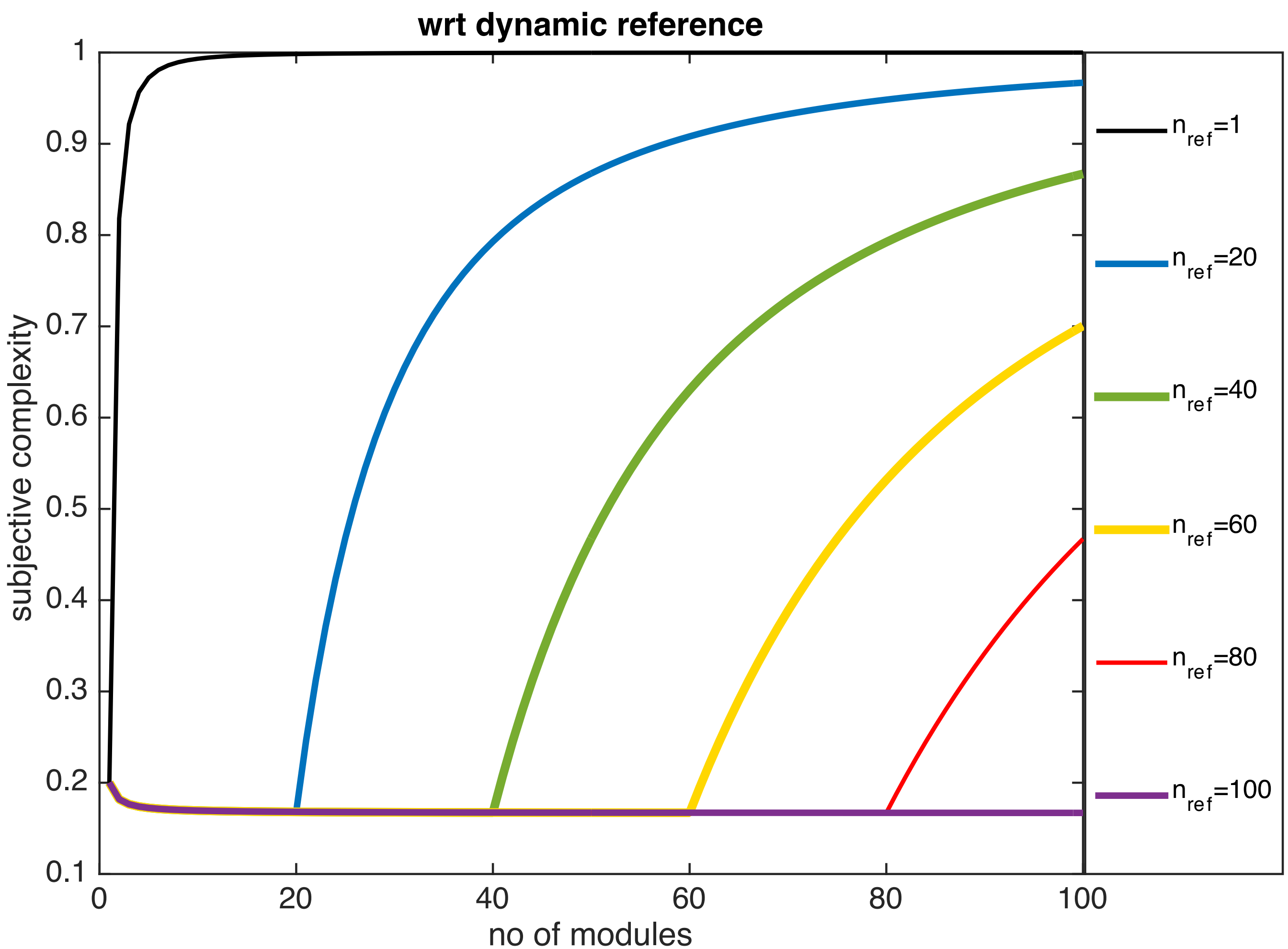
Objective complexity



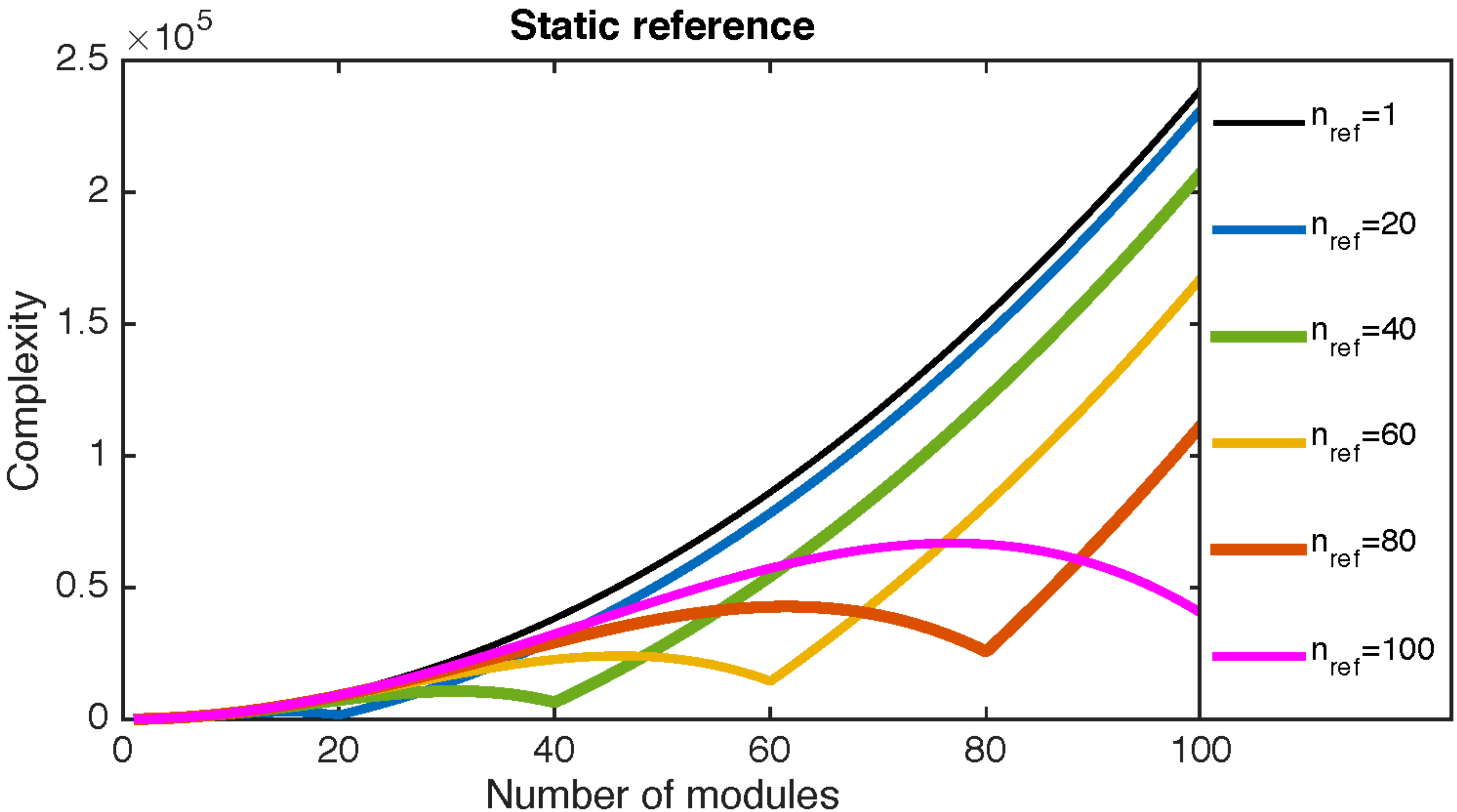
Subjective complexity – static reference



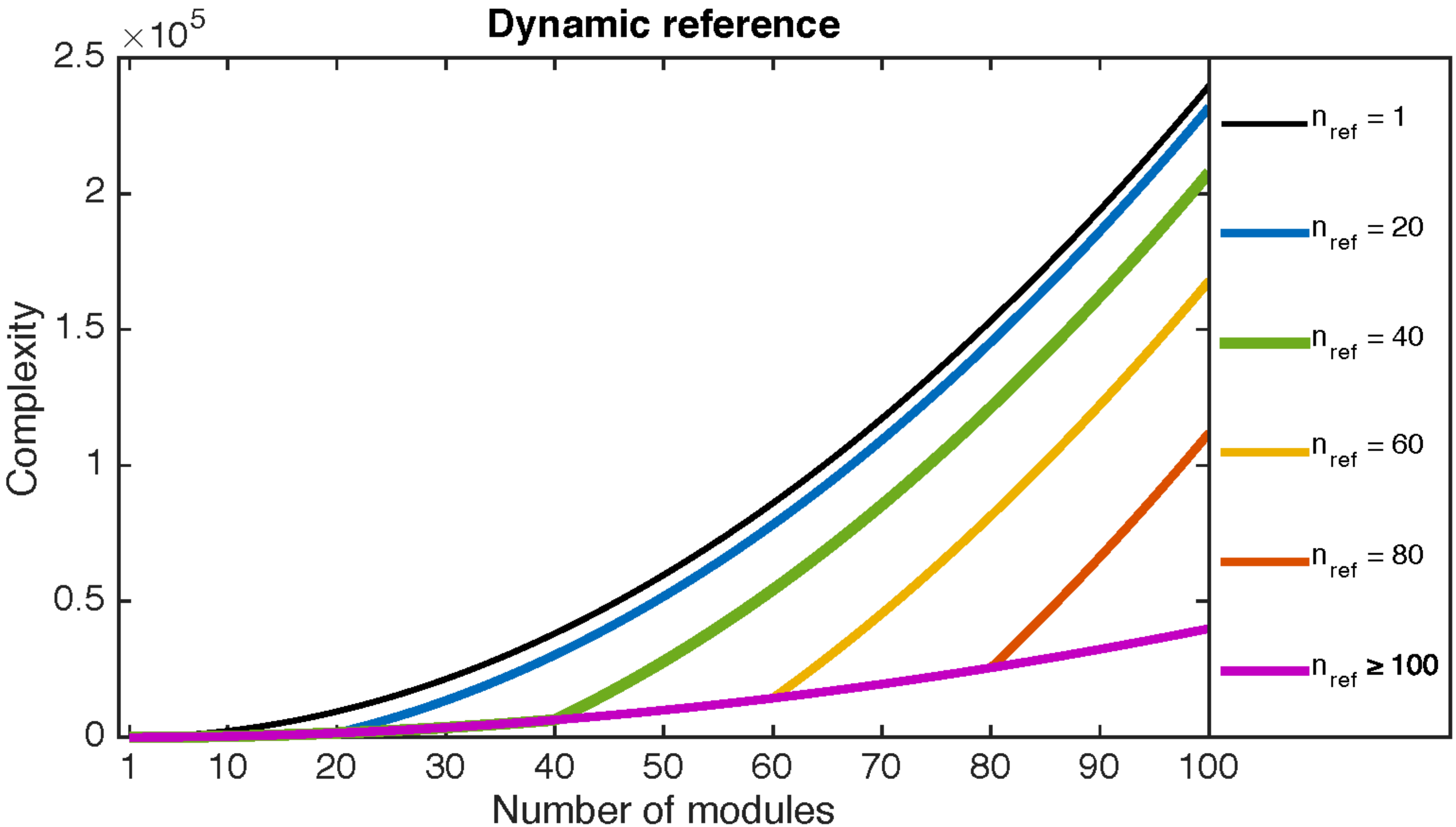
Subjective – Dynamic reference



Resultant complexity - Static



Resultant complexity - dynamic



Conclusion

- Although there are numerous complexity measures in many fields, each tends to be context-dependent within that field.
- We propose a general measure for complexity that accounts for previous measures—Type I, II and III.
- The measure has both objective and subjective elements.
- For systems design:
  - the objective component must be graph theoretic
  - the subjective component is a distance function from a reference for simplicity

25th anniversary annual INCOSE international symposium
Seattle, WA
July 13 - 16, 2015

INCOSE 2015

Fundamentals of System Complexity Measures for Systems Design

Dr Mahmoud Efatmaneshnik
&
Dr Mike Ryan

UNSW AUSTRALIA
Canberra

Capability Systems Centre
UNSW Canberra