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# Exploring System Space with Graph Theory

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# SYSTEMS

EMBEDDED AUTONOMOUS MECHATRONIC CREATIVITY COMPLEXITY  
EMERGENT SYSTEM PHYSICAL  
COLLABORATING CYBER

# Rethinking System Conceptualization



- Interconnectedness and the rise of Cyber Physical Systems
- The National Science Foundation (NSF) describes Cyber-Physical Systems (CPS) as “engineered systems that are built from, and depend upon, the seamless integration of computational algorithms and physical components”
- They tightly intertwine computational elements with physical entities across domains
- The rapid increase in Cyber-Physical Systems is changing the way we develop, manage and interact with systems.
- The NSF notes that CPS challenges and opportunities are both significant and far-reaching. To address these challenges the [NSF is calling for methods to conceptualize and design for the deep interdependencies inherent in Cyber-Physical Systems.](#)

# Systems Engineering Transformation



- While complex systems transform the landscape, the Systems Engineering discipline is also experiencing a transformation - to a model-based discipline.
- While Model Based Systems Engineering (MBSE) shows significant promise, it's still in a formative stage and very few subject matter experts understand formalized systems languages or have ready access to MBSE related tools.
- Key enablers to managing the complexity of systems
  - Applying MBSE to provide explicit integrated system models
  - Expressing system models to deepen our understanding
  - Leverage methods to reach the larger community of stakeholders

# Model Expression



- The Systems Modeling Language (SysML) has proven to be a significant enabler to advance MBSE methods given its flexibility and expressiveness.
- The flexibility of the language and advances in tools also permits easy construction of allocation tables and dynamic tabular representations.
- While SysML provides clarity and consistency, unfortunately, the number of people who know and SysML is still relatively small which has led to some criticism and limited widespread acceptance.
- To bring the full power of MBSE to the larger community system models in SysML can also be represented in a more intuitive form. Not as a replacement to the rich detail provided by the SysML but as a complementary product to conceptualize and design for the deep inter-dependencies inherent in Cyber-Physical Systems.

# Model Expression and Learning



- The objective of model expression is to maximize our ability to translate system data and information into knowledge we can use to improve the trajectory of our programs.
- To improve program and system performance we need to deepen the understanding of system models for the larger community of development stakeholders.
- To ensure we can extend the power of MBSE and SysML to a much larger community it's worthwhile to first consider how we think and interact with information.
  - “Vision trumps all other senses. We are incredible at remembering pictures.”<sup>8</sup>
  - Research has shown repeatedly that we are wonderful at encoding images but not especially good with arbitrary information
  - Our brains are designed for spatial information and our image recognition is very durable.

Pretend your life depends upon remembering the sequence of numbers I am about to share with you

- Do not write them down
- Your results will not be scored
- Everyone passes

Each number will be displayed in a green circle.

#

Please commit to memory the numbers which follow

24

1

5

20

4

25

16

6

10

2

21

22

Write down as many as  
you can remember

How well did you do?

How about a different view?

24

1

5

20

4

25

16

10

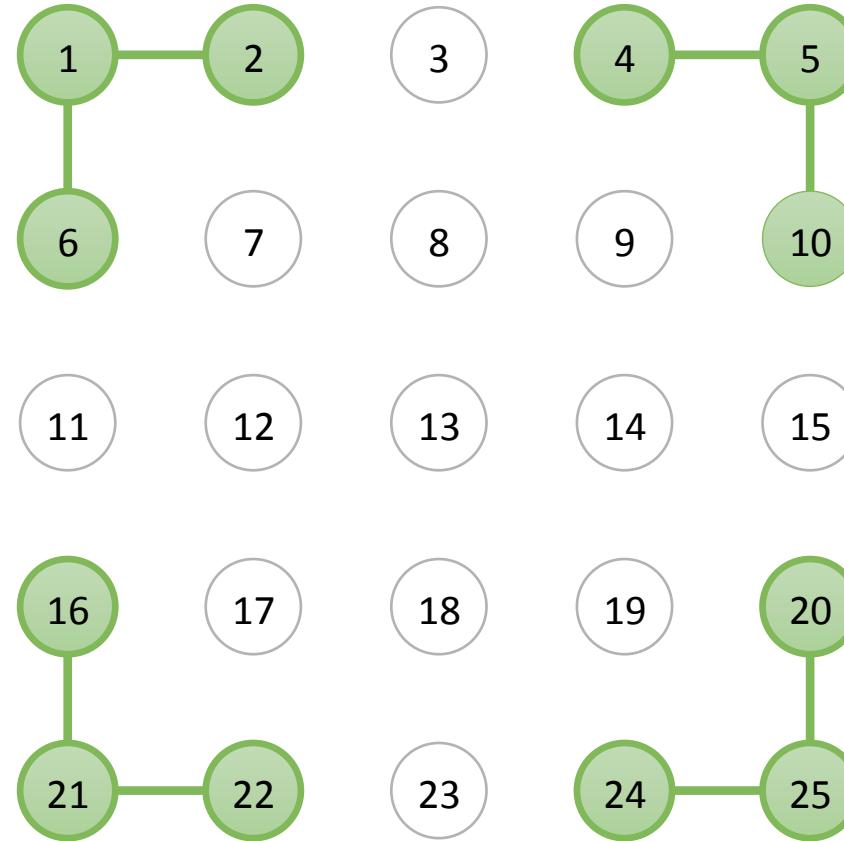
2

21

22

6

Any better?



Could you reproduce the numbers now?

How would you do it?

Do you think of the numbers or the pattern?

# Information Encoding and Allocations



- Just as a computer needs information coded properly in bits – we need images.
- Long strings of numbers are very easy for a computer to recall but spatial information and associations can be far more challenging for a computer. This is true even with the great strides made in machine learning and artificial intelligence.
- As our developments become more digital we need to appropriately allocate activities.
  - Let machines handle what they do well, for instance, storing and reproducing information
  - Focus our attention on leveraging our amazing cognitive ability to compare, contrast, associate, integrate and synthesize information

# Model Expression



- Use of different representations of the same information is not a new concept. Architects, engineers and others often provide multiple views of the same elements to provide users of their products as much clarity as possible.
- To represent form engineers often use left, right, top, bottom, exploded, isometric, cut-out and perspective drawings.
- To represent function we also use several views; this notion is a key tenant of SysML, UML, DODAF and other languages and frameworks.
- In particular, SysML defines nine diagrams, all of which are useful and depict important information about the structure and behavior of a system.

# The Larger Stakeholder Community



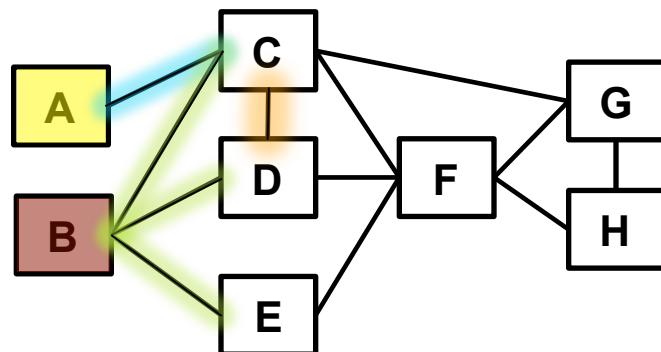
- Many SMEs are required to engineer systems.
- Each SME has tools, languages and methods that they use to model and design systems.
- These languages are often not natural or intuitive to others outside their domain.
- We need to have representations which bridge over roles, domains and areas of functional expertise.
- Graph Theory can provide a means to reach this larger community without significantly sacrificing the power and expressivity of SysML's semantics.
- It can also expose us to new ways of viewing, analyzing and understanding the complex systems we design. Coupled with dynamic visualization we can explore, query and learn the model

# Graph Theory Overview

- The application of graph theory has proven very effective in the design, analysis, management, and integration of complex systems.
- More specifically, it enables the user to model, visualize, and analyze the interactions among the entities of any system.
- Derivatives of Graph Theory, such as Network Analysis and Design Structure Matrix (DSM), are enabled by and support the application of Model Based Systems Engineering (MBSE).
- Both DSM, as a matrix-based system modeling representation, and Network Analysis, as a graphical node and line representation, can be generated from SysML models.
- These representations offer a complementary way to visualize and analyze systems models.

## A powerful paradigm and method to analyze systems

- The diagrams below provide two different views of a generic system with relationships as shown
- For systems, relationships may be interactions of force, information, energy or mass flow
- These diagrams can be powerful in providing understanding of how systems elements interact



**Network View**

Lines indicate connectivity between elements

	A	B	C	D	E	F	G	H
A					X			
B				X	X	X		
C	X				X		X	X
D		X	X			X		
E		X				X		
F			X	X	X		X	X
G			X		X			X
H						X		

**Matrix View**

X's indicate connectivity between elements

*The network view is intuitive and good for understanding very large data sets  
The matrix view provides a compact visual and enables holistic systems modeling*

# Design Structure Matrix Overview

## Design Structure Matrix (DSM)

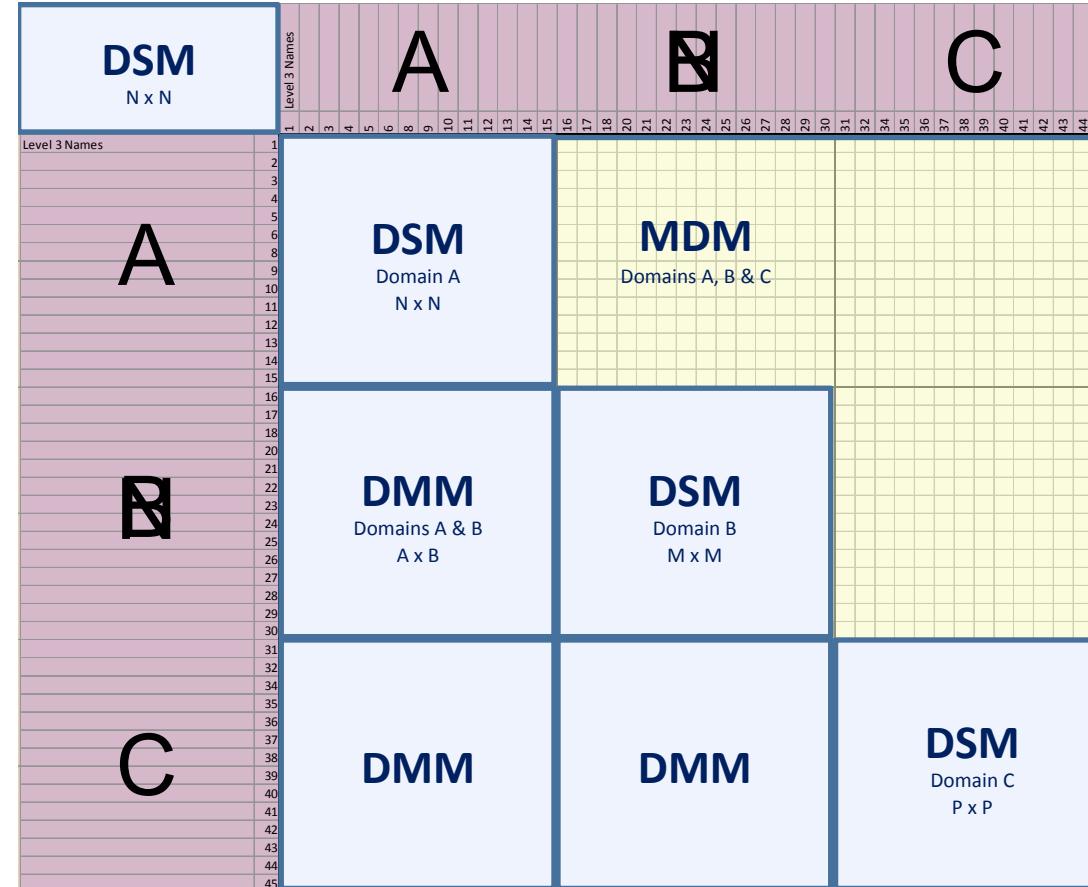
- Square matrix-  $N \times N$  or  $N^2$
- Analyze dependencies within a domain
- Used for products, process and Organizations
- Binary marks ("1" or "X") show existence of a relation
- Numerical entries are weights of relation strength
- Can be directed or undirected (symmetrical)

## Multi Domain Matrix (MDM)

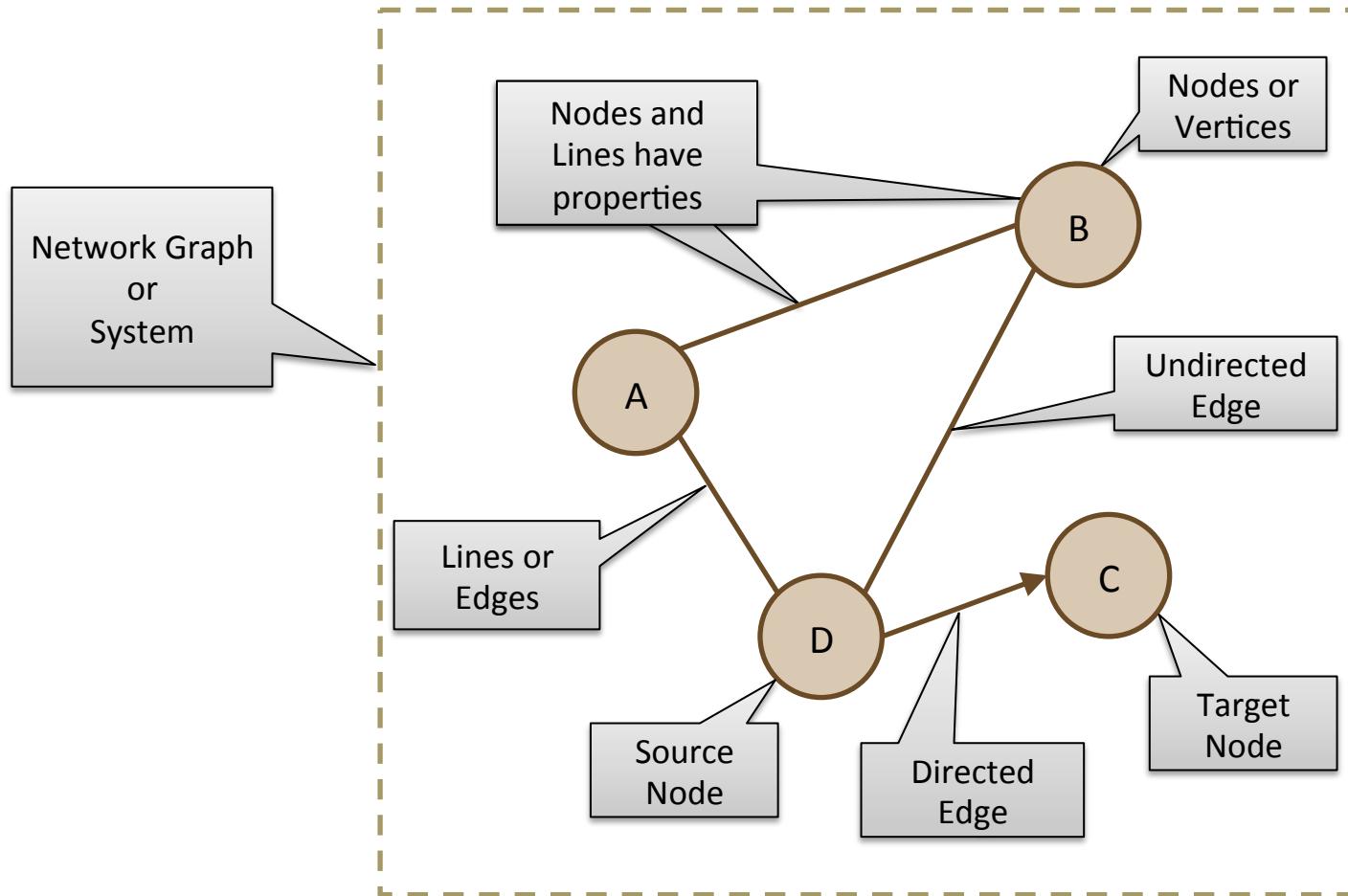
- Square matrix -  $N \times N$  or  $N^2$
- Analyze dependencies across domain
- Combination of DSMs and DMMs
- Especially helpful for DSMs  $> 1000$  elements

## Domain Mapping Matrix (DMM)

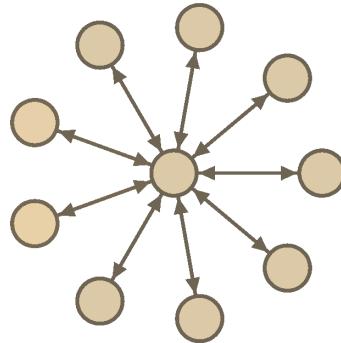
- Normally rectangular matrix –  $N \times M$
- Mapping between two domains



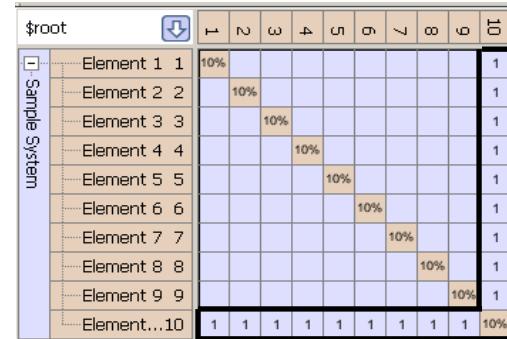
# Graph / Network Overview



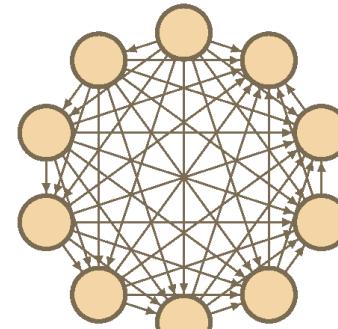
# Graph and DSM Patterns



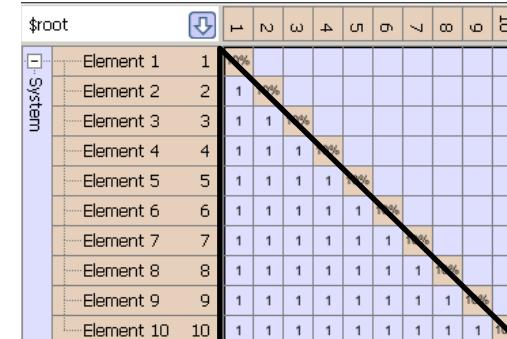
Layout: Concentric



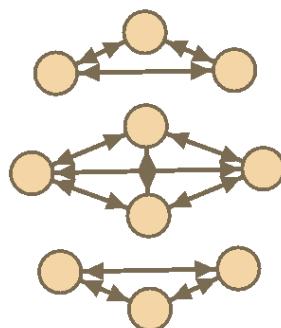
- Symmetrical
- Layered System – Every system uses and feeds element 10



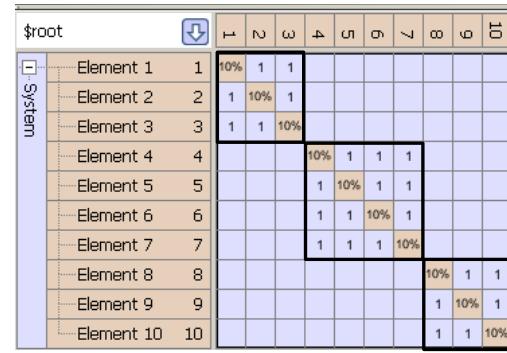
Layout: Circular



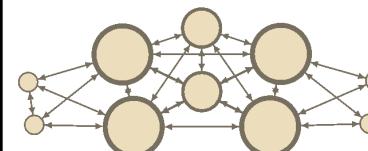
- Non symmetrical
- Layered System – every system uses or feeds every system below it



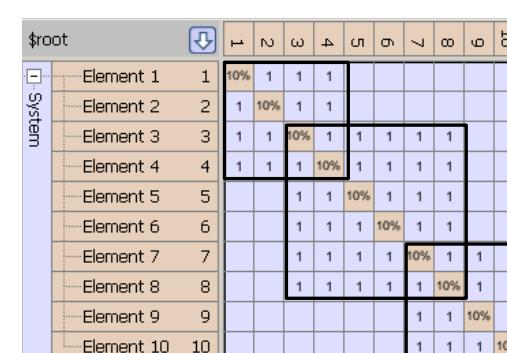
Layout: ForceAtlas2



- Symmetrical
- Non-Overlapping clusters



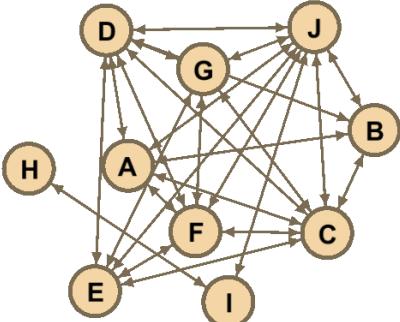
Layout: Yifan Hu



- Symmetrical
- Overlapping clusters

# Graph and DSM Patterns and Algorithms

## Unorganized



\$root	1	2	ω	4	5	φ	7	φ	9	10
System	Element D	1	10%	1	1	1	1	1	1	1
	Element A	2	1	10%		1		1	1	1
	Element H	3		10%		1				
	Element F	4	1	1	10%		1		1	1
	Element I	5		1		10%			1	
	Element E	6			1		10%		1	1
	Element B	7	1	1			10%	1	1	
	Element J	8	1	1	1	1	1	10%	1	1
	Element C	9	1	1	1	1	1	1	10%	1
	Element G	10	1		1		1	1	1	10%

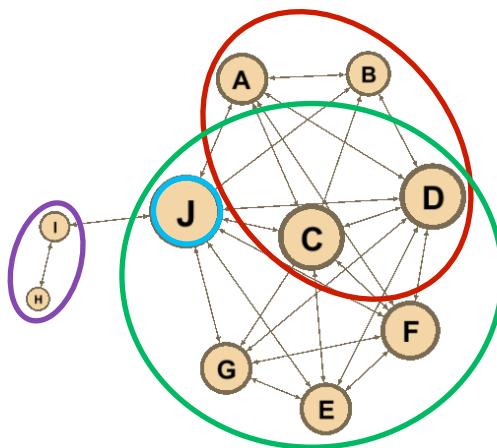
## Network Graph

- Randomly generated

## DSM

- Randomly ordered

## Organized



\$root	1	2	ω	4	5	φ	7	φ	9	10
System	Element H	1	10%	1						
	Element I	2	1	10%						1
	Element A	3		10%	1	1	1		1	1
	Element B	4		1	10%	1	1			1
	Element C	5		1	1	10%	1	1	1	1
	Element D	6		1	1	1	10%	1	1	1
	Element E	7			1	1	10%	1	1	1
	Element F	8			1	1	1	1	10%	1
	Element G	9			1	1	1	1	1	10%
	Element J	10		1	1	1	1	1	1	10%

## Network Graph

- Nodes sized by degree
- Arranged by cluster

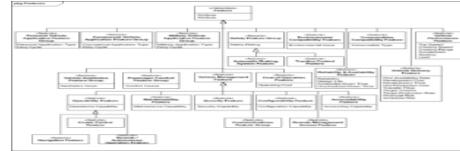
## DSM

- Layered
- Change propagator, Element J, clearly shown at the bottom
- Clustered, showing both overlapping non-overlapping and clusters

# MDM for Hybrid Vehicle – PBSE Tutorial

## Vehicle MDM

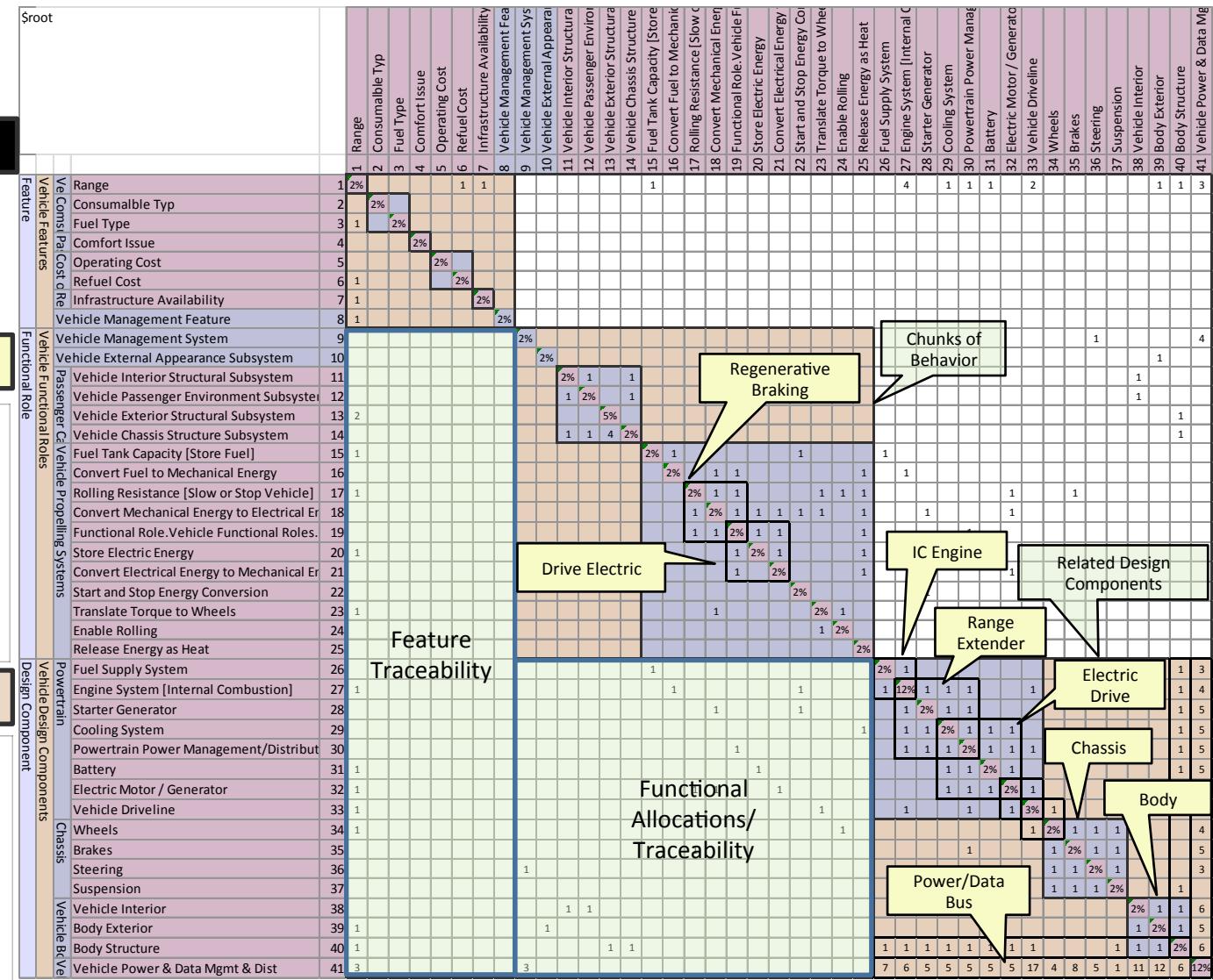
### Features



### Functional Roles

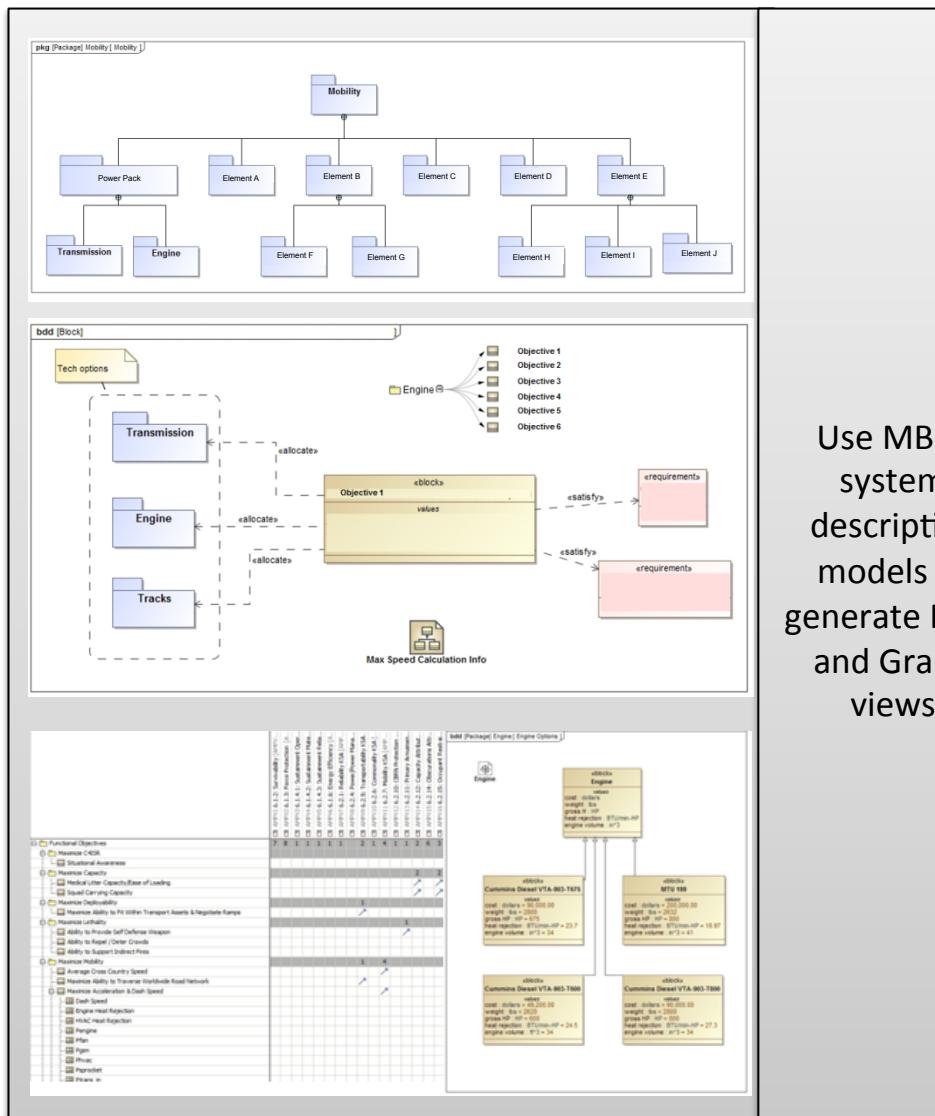


### Design Components



# Models Expressed in SysML, DSM and Graphs

## SysML Models to Graph and DSM Representations

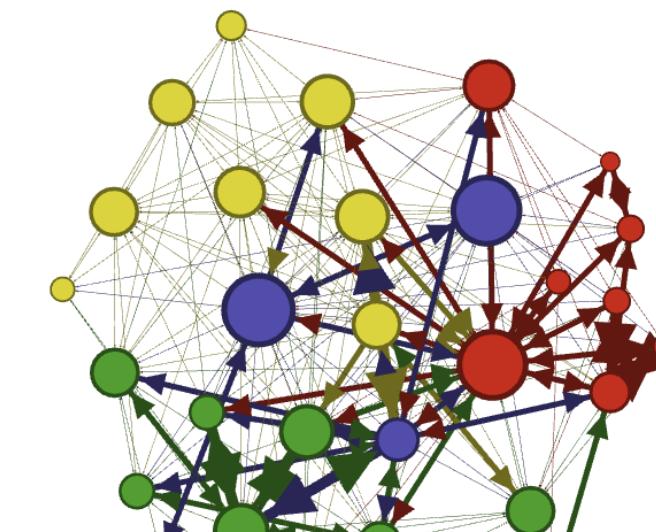


Use MBSE  
system  
descriptive  
models to  
generate DSM  
and Graph  
views

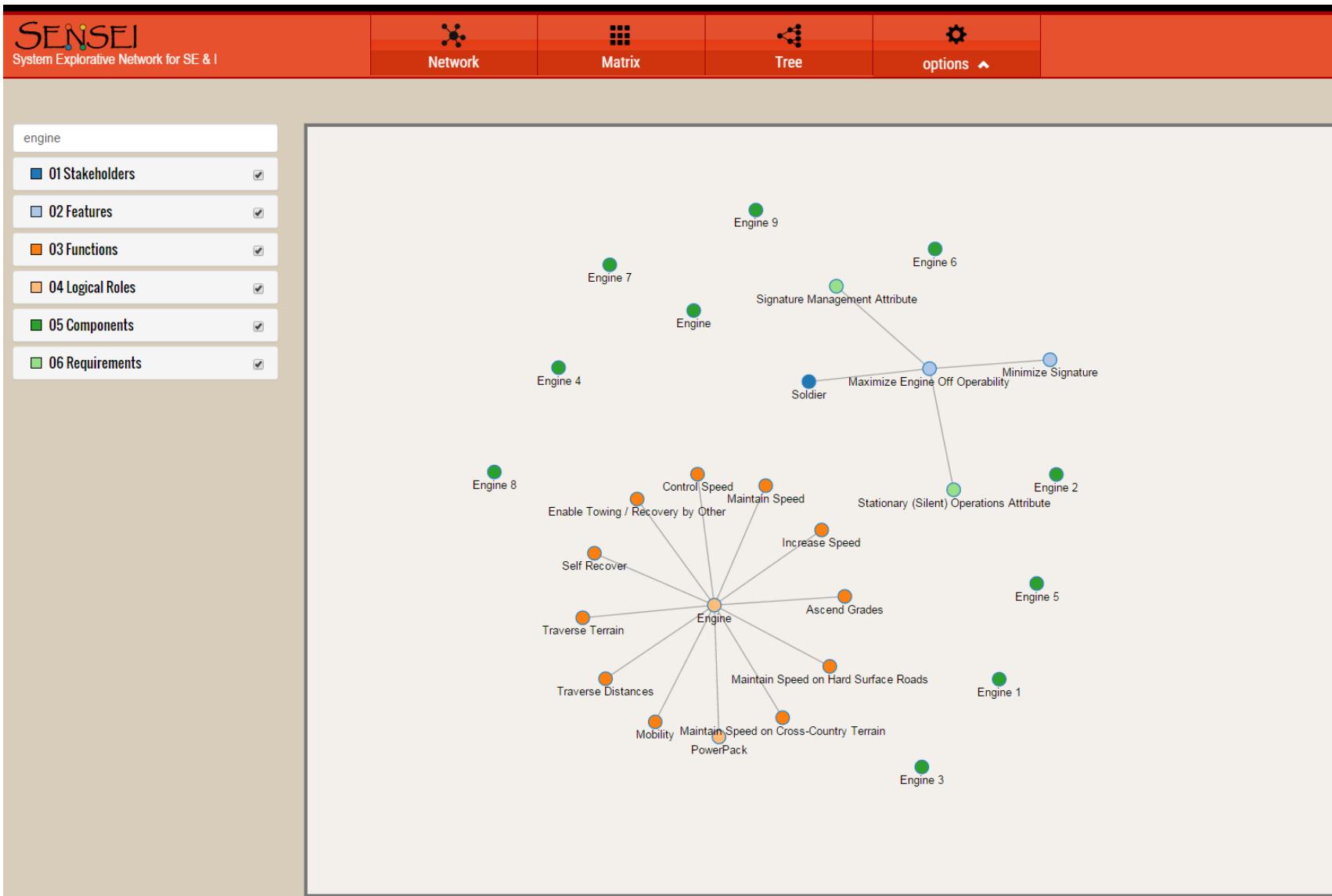
Owner	System	Product Structure Element																																# IF Count	# IF C + IF T C
		3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32				
SubSyst1	System Element 1	3	2																															3	2
SubSyst1	System Element 2	4	2	2	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst1	System Element 3	5	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst1	System Element 4	6	5	2	1	1	1	1	1	1	2	3	1	2	2	2	2	2	3	2	2	2	6	SE 11	13	19	4	23	13	19	4	23	13		
SubSyst2	System Element 5	7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 6	8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 7	9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 8	10	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 9	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 10	12	1	2	1	1	1	1	1	2	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 11	13	1	1	3	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst2	System Element 12	14	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	
SubSyst3	System Element 13	15	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 14	16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 15	17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 16	18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 17	19	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 18	20	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 19	21	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 20	22	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 21	23	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst3	System Element 22	24	2	1	1	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 23	25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 24	26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 25	27	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 26	28	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 27	29	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 28	30	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 29	31	1	1	2	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
SubSyst4	System Element 30	32	2	1	2	1	2	2	2	1	1	1	2	1	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

### Dependency Kind

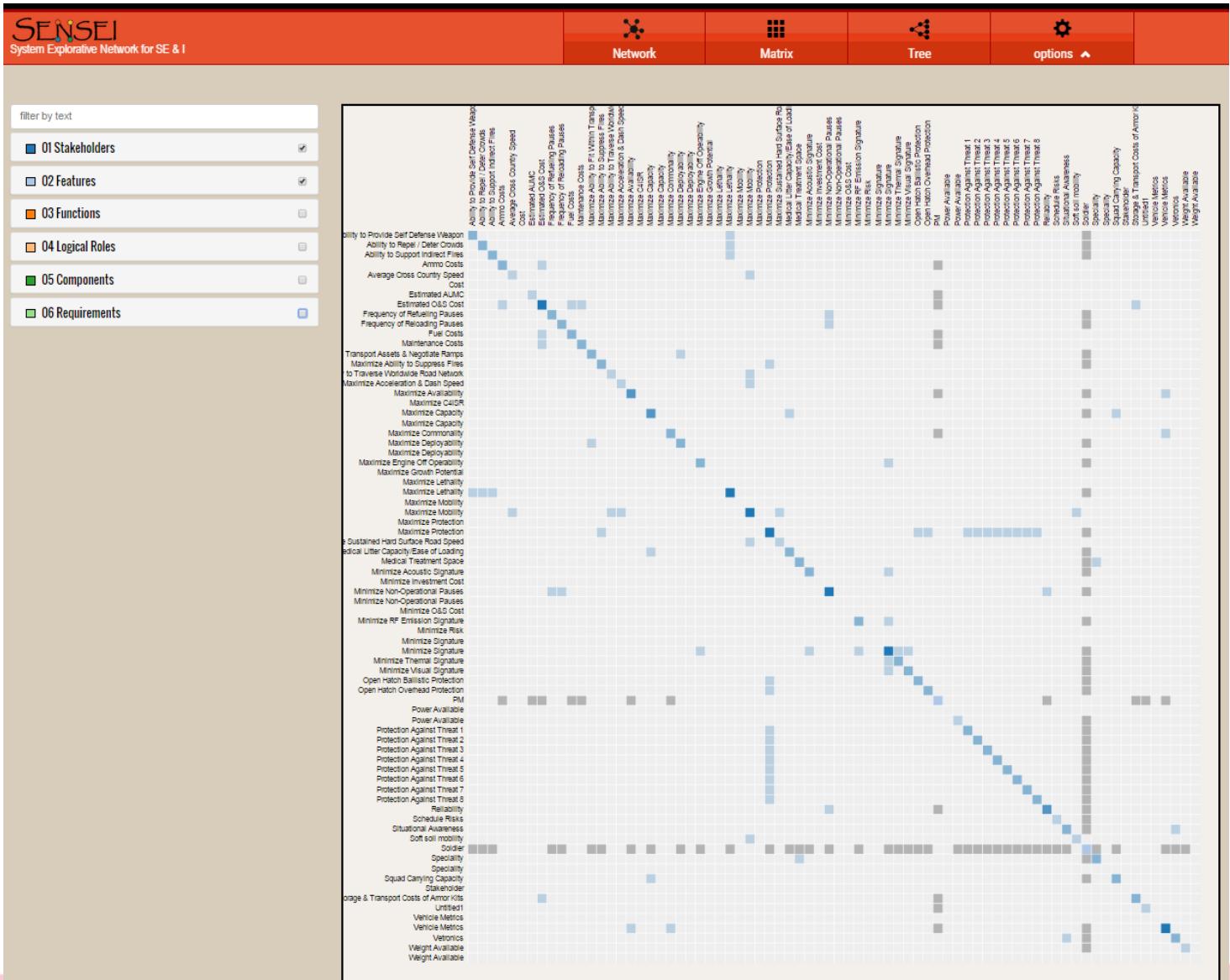
- Mass Flow (5%)
- Force (24%)
- Energy (26%)
- Data (45%)



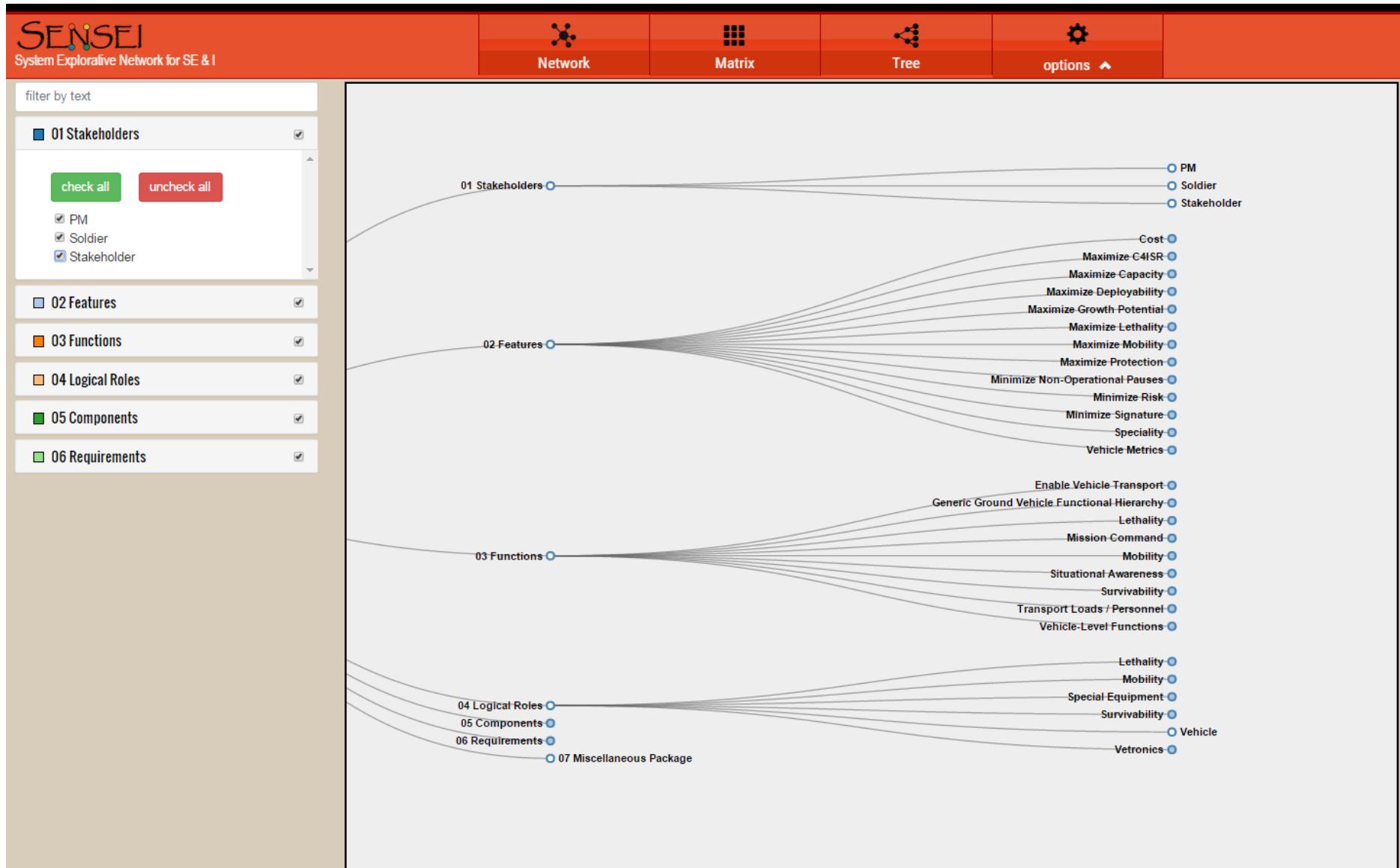
# Dynamic Visualization Network



# Dynamic Visualization Matrix



# Dynamic Visualization Tree



# Conclusions



- As systems become more and more complex they provide both incredible opportunity and risk.
- The benefits of Model Based Systems Engineering approaches are a powerful way to model, understand and manage the evolution of systems.
- Translating detailed models into simple system representations (graphs and matrices) and providing dynamic visualizations can extend the full power of model based methods to the larger engineering community and deepen our collective understanding
- When coupled with an understanding of how we learn; models provide an excellent opportunity to help development teams and leadership gain insights, build intuition and speed the innovation

***"We can't solve problems by using the same kind of thinking we used when we created them."***

Albert Einstein

# Troy Peterson Bio



## Troy Peterson

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Vice President & Fellow  
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313.806.3929

Troy Peterson is a Vice President with System Strategy, Inc. A consulting firm instituting model based capabilities to manage complexity and speed innovation. As a Fellow,

Troy has led several international projects and large teams in the delivery of complex systems. His experience spans commercial, government and academic environments across all product life cycle phases. Recent engagements include Contingency Basing, the Ground Combat Vehicle (GCV), Mine Resistant Ambush Protected (MRAP) vehicle and developing engineering capability within organizations responsible for research, development, acquisition and system of systems engineering and integration.

Troy's impact has led to his appointment to six different boards to improve engineering education and method application. He frequently speaks at leading engineering conferences and was recently appointed by INCOSE as the lead for transforming Systems Engineering to model based discipline.

Prior to joining System Strategy, Inc. Troy was a Booz Allen Fellow and Chief Engineer. He worked at Ford Motor Company and as an entrepreneur operating a design and management consulting business. Troy received his B.S. in Mechanical Engineering from Michigan State University, his M.S. in Technology Management from Rensselaer Polytechnic Institute, and an advanced graduate certificate in Systems Design and Management from the Massachusetts Institute of Technology (MIT). He holds INCOSE Systems Engineering, PMI Project Management, and ASQ Six Sigma Black Belt certifications.

## Dynamic Visualization of Complex Systems: Extending the Impact of Model Based Systems Engineering

As today's cyber physical systems become more and more complex they provide both incredible opportunities and risks. In fact, rapidly growing complexity is a significant impediment to the successful development, integration, and innovation of systems. Over the years, methods to reduce system complexity have taken many forms. Model Based Systems Engineering (MBSE) provides organizations a timely opportunity to address this complexity. MBSE tools, languages and methods are still in a formative stage and continue to evolve. The Systems Modeling Language (SysML) has proven to be a significant enabler to advance MBSE methods given its flexibility and expressiveness. The flexibility of the language and advances in tools also permits easy construction of allocation tables and dynamic tabular representations. While these strengths provide clarity and consistency unfortunately the number of people who know and can read SysML well is a relatively small group. This has led to some criticism and has limited widespread acceptance. To bring the full power of MBSE to the larger community system models represented in SysML can be represented in a more intuitive form for the larger community without sacrificing the power and expressivity of SysML's semantics. More specifically JavaScript libraries for providing dynamic interactive visualizations can provide the larger community insights into these powerful system models. These representations can help teams gain insights, build intuition and ultimately help speed the innovation process. This briefing will share how the translation of SysML models into Data-Driven Documents (D3.js or D3) provides a rich representation and understanding of system models previously only provided to limited set system modelers. An example of a system model within a well-known MBSE tool will be translated into D3 graph, matrices and dendrogram/tree representations. These views can provide teams a simple yet powerful means to analyze and manage complex systems. More specifically, they enable the larger community visualize, and analyze the key relationships represented in system models.