27th annual INCOSE international symposium
Adelaide, Australia
July 15 - 20, 2017

ENGINEERS AUSTRALIA SESA

Control Theoretic Modeling and Simulation of Supply Chain Management System

Author
Dr. Obaid Ur Rehman
A/Prof. Mike J. Ryan

Capability Systems Centre
UNSW CANBERRA

Introduction

- The supply system is a dynamical system with a range of lead times and varying delays between supply nodes.
- The demand and supply of such a system vary with time and require an effective distribution and management policy to deal with all variations.
- In a real-life, the nominal demands are usually known, however, difficulty arises when the uncertainty in the demand makes it difficult to maintain the stability of the system.

Image credit *pixabay.com*

Introduction

- Supply chain systems are multidimensional and there are several factors which affect the system stability.
- In order to systematically address this problem, researchers have proposed various dynamical modelling and simulation approaches.
- The modelling and simulation approach in this paper is based on the control theory approach where ordering policy is determined through feedback which is a function of the system states and demand from customer.

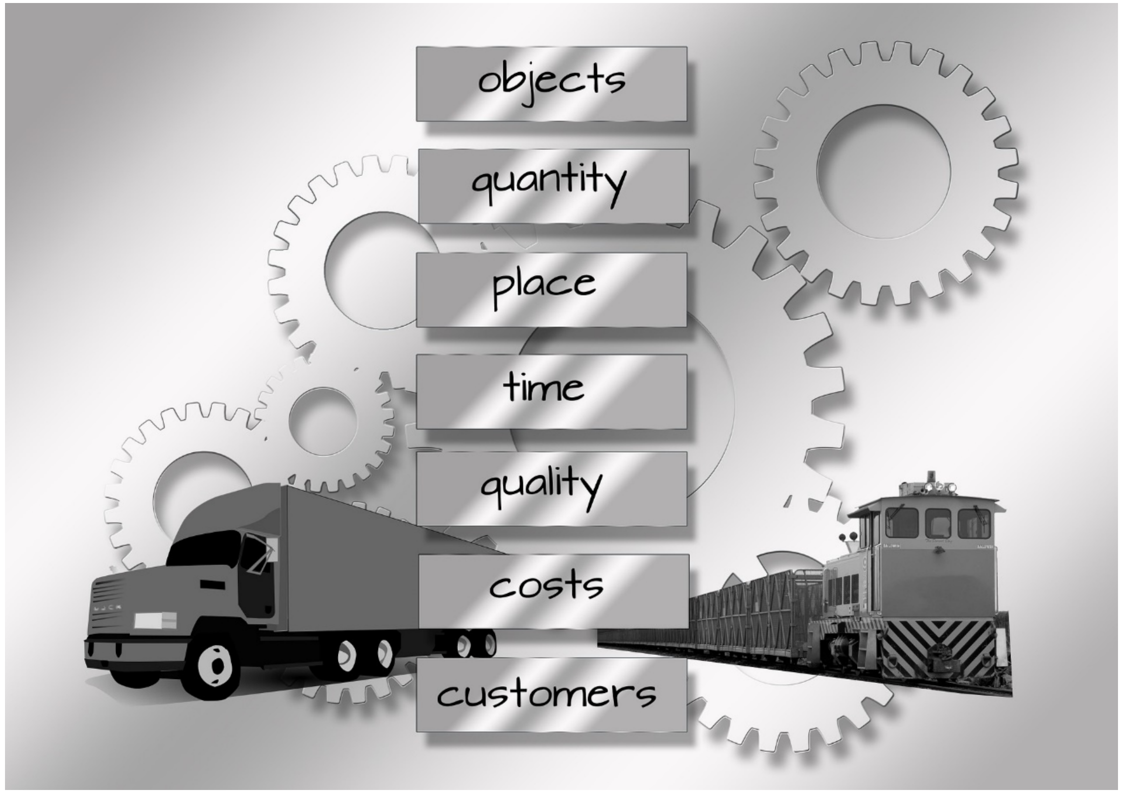


Image credit *pixabay.com*

Outline

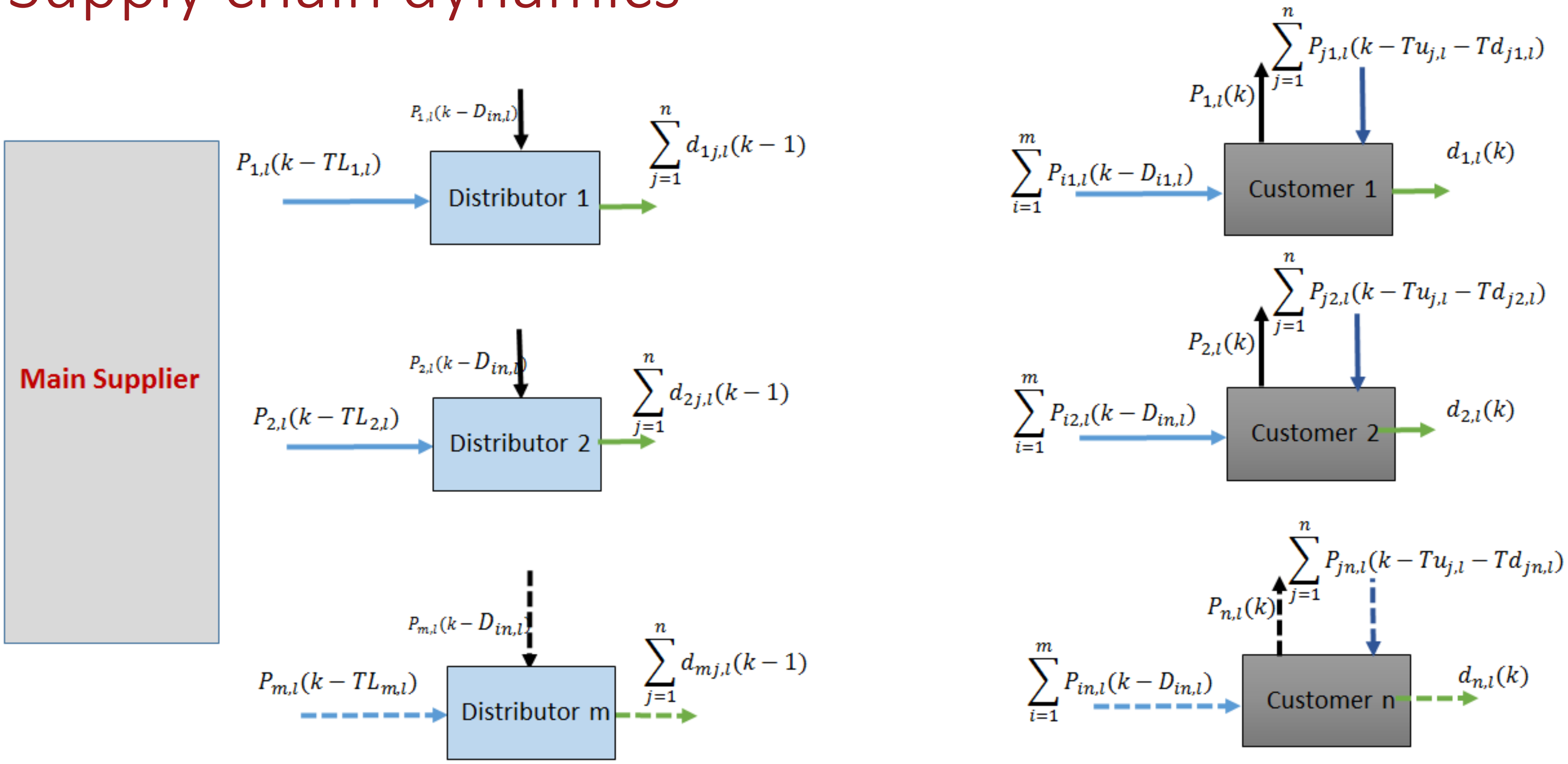
- Problem statement.
- Control theoretic model.
- System simulation model.
- Simulation results.
- Conclusion.

Problem Statement

- Distribution of several packages to customers at several locations managed by a Prime Service Contractor (PSC).
- There are n customer locations, m distributor locations, and l types of packages.
- A customer requests packages and PSC considers all the available resources at different locations and chooses an optimum ordering policy as well as the most cost-effective delivery route depending on the time required to deliver, and the cost of delivery.
- Each location has different lead times for the delivery.
- Packages are re-usable.

The main objective is to help operation managers to obtain cost-effective ordering and resource routing policies such that system remains stable.

Supply chain dynamics



Dynamic Model

- Stocked package dynamic equation for customer location:

$$S_{p,l}(k)=S_{p,l}(k-1)+\sum_{j=1,\, j\neq p}^{n} P_{jp,l}(k-Tu_{j,l}-Td_{jp,l}) + \sum_{i=1}^{m} P_{ip,l}(k-D_{in,l}) - P_{p,l}(k)-d_{p,l}(k)$$

- Stocked package dynamic equation for distributor location:

$$D_{q,l}(k)=D_{q,l}(k-1)+P_{q,l}(k-TL_{q,l})+P_{q,l}(k-D_{in,l})-\sum_{j=1}^{n} d_{q,j,l}(k-1)$$

Augmented Dynamics

- Augmented customer dynamics

$$X_{p,l}(k)=A_{p,l}X_{p,l}(k-1)+B_{p,l}U_{p,l}(k)-d_{p,l}(k),$$
$$Y_{p,l}(k)=C_{p,l}X_{p,l}(k-1)+D_{p,l}U_{p,l}(k),$$

- Augmented distributor dynamics

$$X_{q,l}(k)=A_{q,l}X_{q,l}(k-1)+B_{q,l}V_{q,l}(k)$$
$$Y_{q,l}(k)=C_{q,l}X_{q,l}(k-1)+D_{q,l}V_{q,l}(k)$$

- Augmented system model

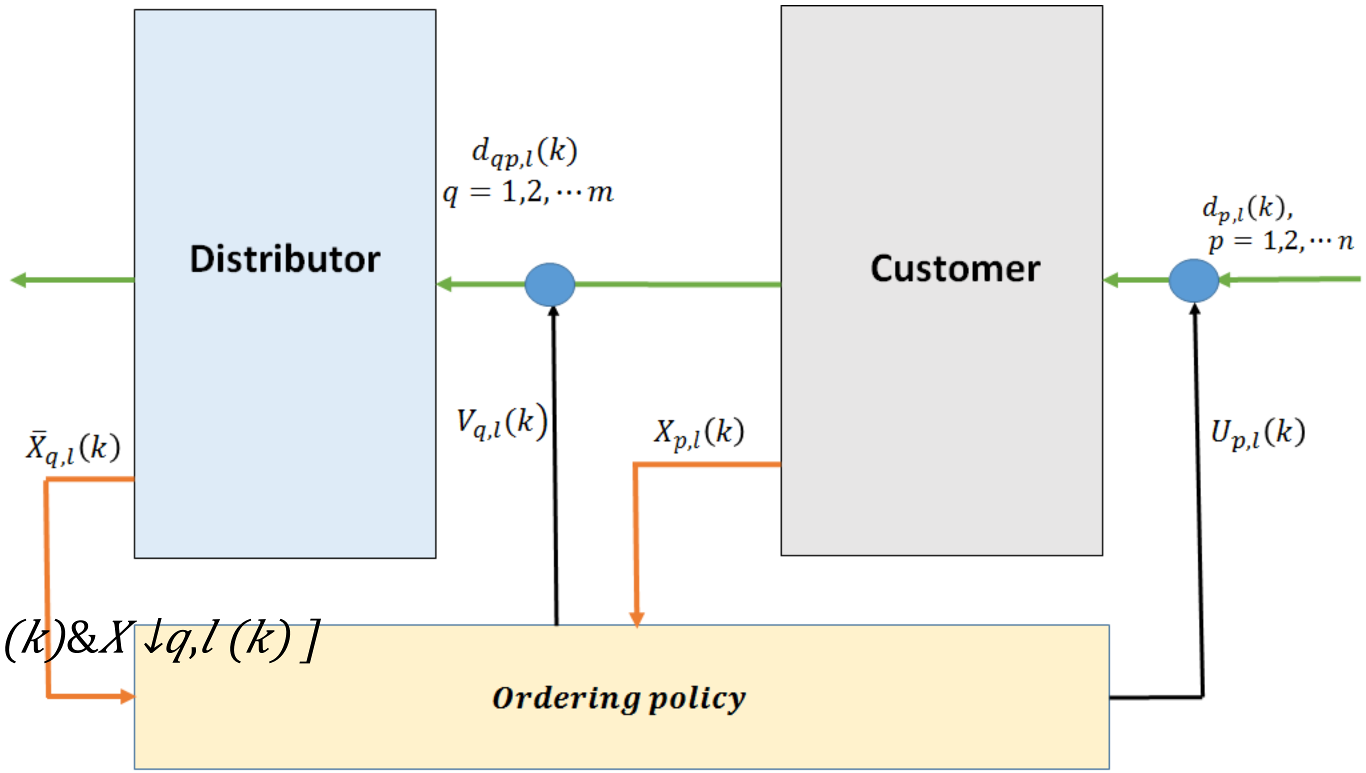
$$\begin{bmatrix} X_{p,l}(k) \\ X_{q,l}(k) \end{bmatrix}=\begin{bmatrix} A_{p,l} & 0 \\ 0 & A_{q,l} \end{bmatrix}\begin{bmatrix} X_{p,l}(k-1) \\ X_{q,l}(k-1) \end{bmatrix}+\begin{bmatrix} B_{p,l} & 0 \\ 0 & B_{q,l} \end{bmatrix}\begin{bmatrix} U_{p,l}(k) \\ V_{q,l}(k) \end{bmatrix}$$

State feedback ordering policy

- The ordering policy can be obtained by designing a state feedback stabilizing controller using approaches from the control systems theory literature.

- There is a rich literature available for designing a stabilizing controller for dynamical system.

$$\begin{bmatrix} U_{p,l}(k) \\ V_{q,l}(k) \end{bmatrix} = K_{order} \begin{bmatrix} X_{p,l}(k) \\ \bar{X}_{q,l}(k) \end{bmatrix}$$



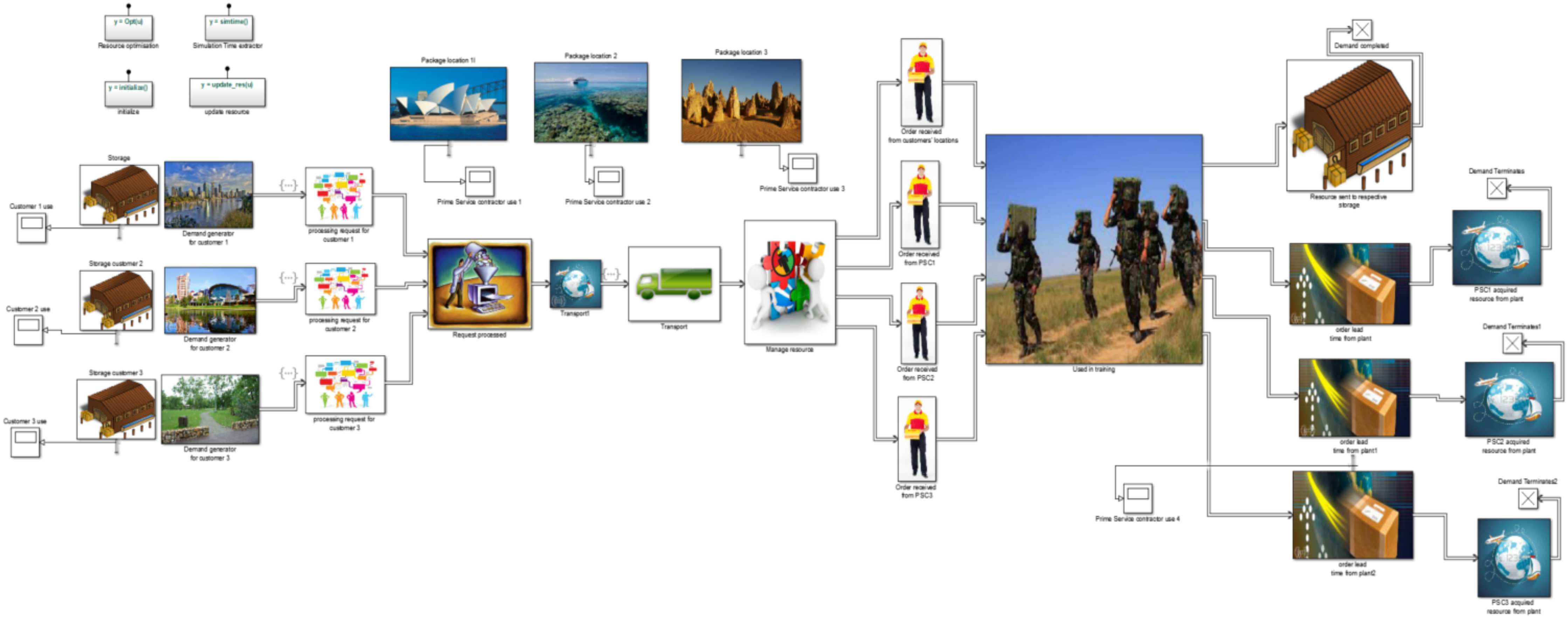
Simulation Model

- The discrete time model is used to design a simulation model for the complete supply chain.
- In this work, Simulink® toolbox SimEvents® from MathWorks® has been used to design a simulation model using the dynamic model presented in the previous slide.
- The simulation model simulates various scenarios for the supply chain problem and allows operational managers to systematically order and route the resources based on the expected future orders.
- The primary benefit of the proposed simulation model
  - it can be used to determine initial resource allocation,
  - and optimum routing policy to keep the system stable by simulating a number of possible scenarios under uncertainties.

Model Attributes

- Demand originated at the customer locations i.e. demand rate and quantity, and transportation time are considered as a random variable assuming Gaussian distribution.
- The model also considers cost of the operation, i.e.
  - *Cost of Storage.*
  - *Cost of Transport*.
  - *Cost of failure to meet order time.*
  - *Cost of purchase of packages from main supplier.*

Simulation Model

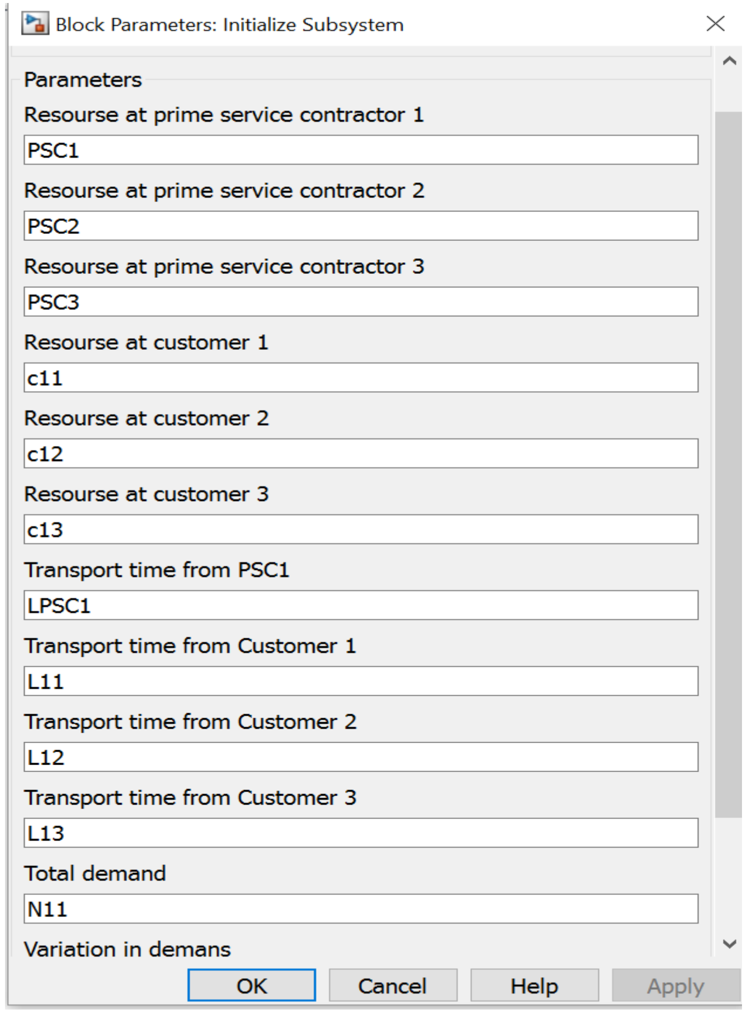


Simulation operation

- Once the demand appears at the customer location, it is processed by the PSC which determines the quantity required from different locations based on meeting the demand with minimum cost and the required delivery (order) time.
- when packages are arrived and utilized at the customer locations,
  - it could remain at the customer location for any future need
  - or may be returned to the nearest distributor location according to the given policy.
- These two cases are compared in the optimisation model and the minimum cost case is implemented by considering.
  - the cost and time of returning and storing to the distributor locations
  - storing at the current customer location.

Simulation initialisation

- Current resources available at the distributor and customer locations.
- Transportation time from one location to another location.
- Total quantity of packages required.
- Required delivery time.
- Cost of holding packages at different locations.
- Uncertainty level in demand rate.
- Number of packages required and their use time.
- Simulation run time.



Simulation Results

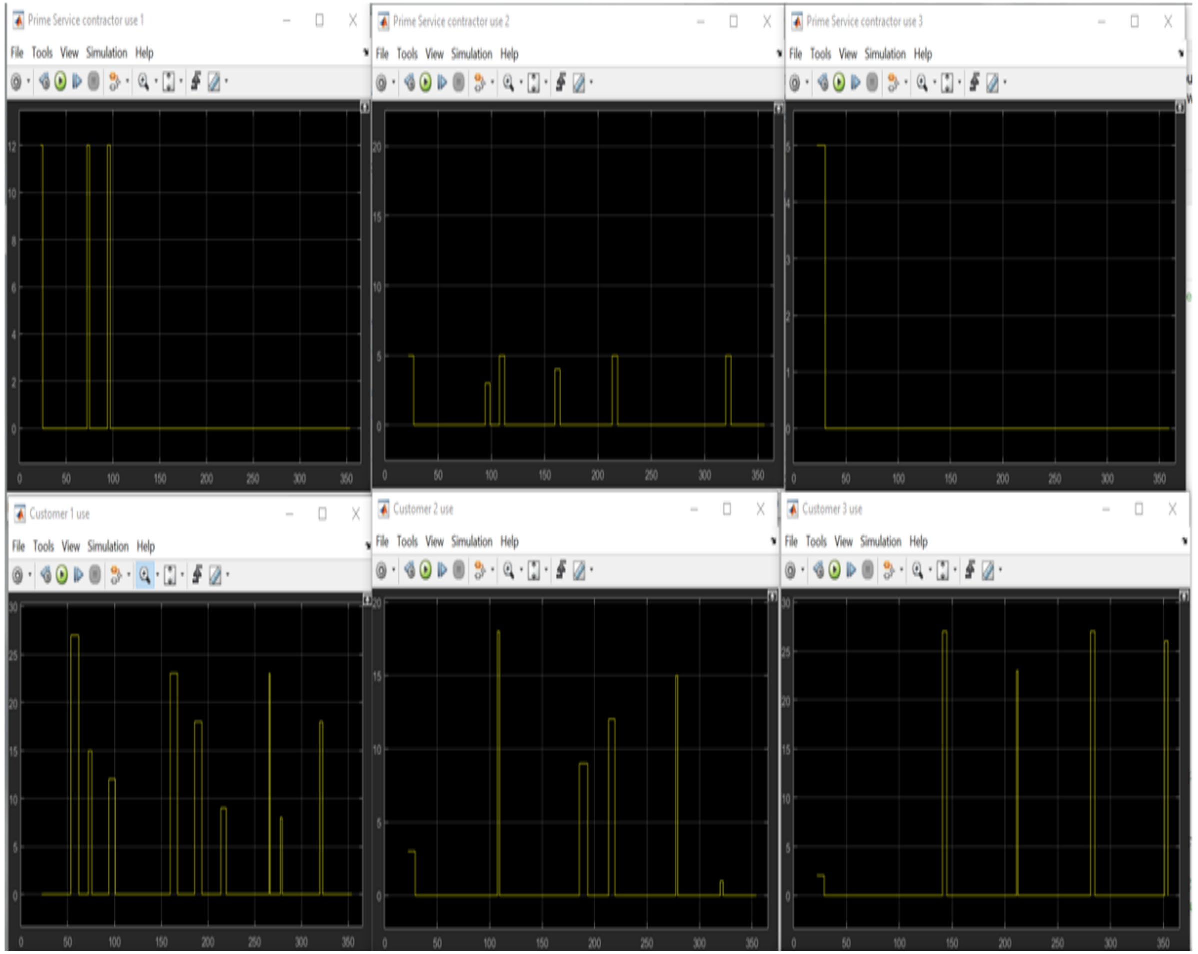
- The result shows the demand and supply of packages during one year
- The result can be seen in two output formats:
  - Matlab output screen.
  - Simulink graphical viewer.
- The Matlab screen output shows the amount of current packages available at different locations and the transportation time to meet the demand.
- The Matlab screen output also shows the optimum ordering policy i.e. the number of packages required from each location.
- The Simulink output shows the number of packages supplied from each location along with their usage time.

Matlab Screen Output

```
Demand 1= 27 units for customer 1 on day 0 to use for 7 days
Current resource available
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    10     5      5      0      3      2

Current Resources are not enough. Resources are ordered from the main supplier.
Optimum resource demand from each available resource
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     5      5      0      3      2

Transport time = 2
Resources are acquired at day 20 and now sent
Current resource available after order from plant
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     5      5      0      3      2

----------------------------------------------------------------------------------
----------------------------------------------------------------------------------
Demand 2= 27 units for customer 1 on day 53 to use for 8 days
Current resource available
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
   0      0      0     27      0      0

Number of days resource held after using by cutomers
    24

Optimum resource demand from each available resource
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
     0      0      0     27      0      0

Transport time = 0.5
```

```
Demand 1= 27 units from customer 3 on  day 70 to use for 4 days
Current resource available
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     5      5     27      0      0

Optimum resource demand from each available resource
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     0      0     15      0      0

Transport time = 2
----------------------------------------------------------------------------------
----------------------------------------------------------------------------------
Demand 1= 27 units from customer 2 on  day 92 to use for 7 days
Current resource available
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     5      5     12      0     27

Optimum resource demand from each available resource
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     3      0     12      0      0

Transport time = 2
----------------------------------------------------------------------------------
----------------------------------------------------------------------------------
Demand 3= 23 units for customer 1 on day 106 to use for 2 days
Current resource available
PSC1,  PSC2,   PSC3, Customer 1, Customer 2, Customer 3
    12     5      5      0     27     27

Number of days resource held after using by cutomers
   44.5000
```

Simulink Graph Output



- If the results shows that the accumulation of packages becoming high as time progresses at the customer or distributor locations or the delivery time is not met, or it is required to order packages from the main supplier during the operation, then it means that the system may become unstable. Therefore, a revised policy may be needed.
- The results can be can be presented in various ways.

- The simulation result can be customized using different simulation output from Simulation library based on the operational manager preference.

- In addition, a Monte Carlo type simulation using the proposed mathematical model can also be developed to simulate various scenarios together for better risk management under demand and supply uncertainties.

Conclusion

- Developed a linear discrete time control theoratic model of a supply chain with multiple supply nodes and pure delays.
- The model presented here can be used to develop an optimal ordering policy so that the entire system remains stable.
- In addition to the mathematical model of the supply chain, a simulation model is also developed using Simulink® toolbox SimEvents®.
- The simulation model is shown to test optimum ordering and routing policies.
- The simulation model can be used to implement and test and compare various other ordering policies.

27th annual INCOSE international symposium

Adelaide, Australia

July 15 - 20, 2017

ENGINEERS AUSTRALIA

SESA

www.incose.org/symp2017