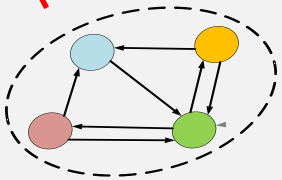
34th Annual INCOSE international symposium

hybrid event

Dublin, Ireland
July 2 - 6, 2024

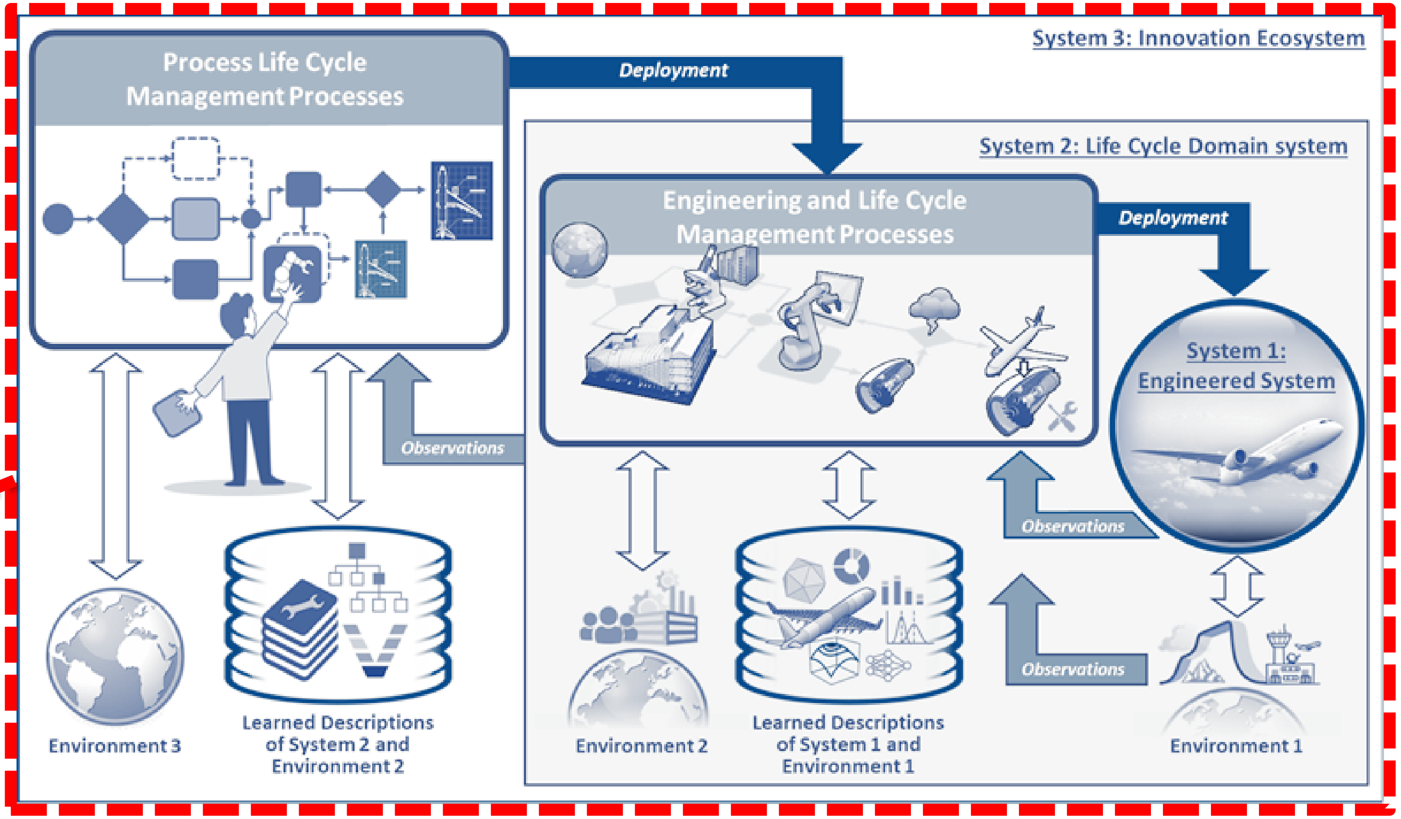


$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = - \frac{\partial H}{\partial q_i}$$

W R Hamilton
Dublin, Ireland
(1805-1865)

INCOSE ASELCM Innovation Ecosystem Pattern



What Can Hamilton Tell Us?

Innovation Ecosystem Dynamics, Value, and Learning I

V1.2.1

Abstract

- Held in Dublin, Ireland, INCOSE IS2024 invites us to refresh understanding of contributions to systems engineering by Ireland's greatest mathematician--Sir William Rowan Hamilton (1805 - 1865), Professor of Astronomy at Trinity College Dublin, and Royal Astronomer of Ireland.
- His profound contributions to STEM deserve greater systems community attention.
- Supporting theory and practice, they intersect Foundations and Applications streams of INCOSE's Future of Systems Engineering (FuSE) program.
- Strikingly, key aspects apply to systems of all types, including socio-technical and information systems.
- Hamilton abstracted the energy-like generator of dynamics for all systems, while also generalizing momentum.
- Applied to the INCOSE Innovation Ecosystem Pattern as dynamics of learning, development, and life cycle management, this suggests an architecture for integration of the digital thread and machine learning in innovation enterprises, along with foundations of systems engineering as a dynamical system.

Contents

Launching the STEM revolution (c. 1700 – 2000)

- Millennia of observation and thought about natural phenomena were punctuated by the much shorter STEM revolution:
  - In less than 300 years, Newton, Lagrange, Gauss, Euler, Jacobi, Hamilton, Gibbs, and others set the conceptual and mathematical framework supporting the dramatic acceleration of STEM.
- What followed rapidly changed the quality, length, and possibilities of human life.
- William Rowan Hamilton made enormous contributions to those theoretical foundations of scientific and engineering disciplines.
- His mathematical patterns describe phenomena of mechanics, electrical science, thermodynamics and subsequent disciplines--the foundations of today's STEM.
- "The chief law of physics, the pinnacle of the whole system is . . . the principle of least action" (Hamilton's Principle)--Max Planck, 1925. [1]
- A surprise: We will summarize here how Hamilton's work also applies to information systems and socio-technical systems:
  - The innovation ecosystem / supply chain itself, viewed as a dynamical system!

Challenges in current innovation ecosystems

A. Project and program planning: What are predicable efforts, times, and costs of performing innovation and life cycle management? What are related uncertainties (and consequent risks) in those predictions? [2-5]

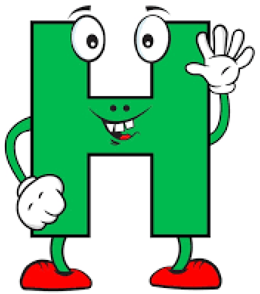
B. Project execution management: As projects are performed (and encounter real-world perturbations only partly predictable), what are the means of preparing, monitoring, and directing them for optimum outcome—including decision-making in particular? During complex multi-enterprise development projects, how can we detect and act on systemic project uncertainties and instabilities threatening success? [6-9]

C. Project learning and its recurrent application: What are means and effects of accumulating new experience in items (A) and (B) above, distilling, managing trust in, and applying knowledge and competency in future projects? [9-18]

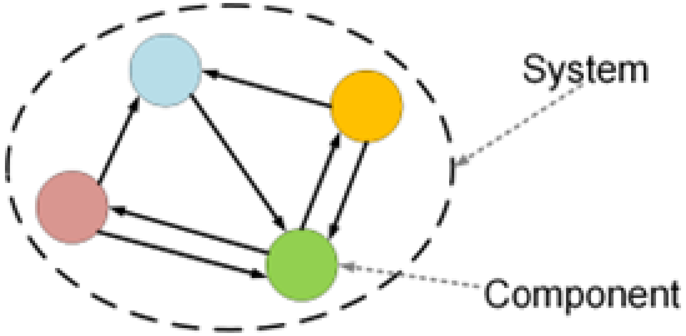
D. Information and information system roles in items (A), (B), and (C) above: A common thread through the above are roles of information and information systems—both those using engineered information technologies and those performed by human beings. What is the theoretical basis for engineering the performance of these subsystems as an integrated part of larger enterprise systems in which they appear? [9, 18-22]

Are current assumptions too conservative?

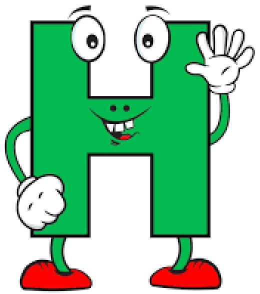
- Hamilton's framework is most familiar to engineers in mechanical, civil, or electromagnetics settings.
- The systems community may be assuming that Hamilton's mathematical contributions do not address the socio-technical and information system questions above in a practical way.
- Symptom: A continued call and search for what are perceived as missing theoretical foundations for the science and engineering of generalized systems. [24]
- Disciplines in engineering and sciences are concerned with phenomena (e.g., mechanical, electrical, chemical) specific to those disciplines, leading to impactful phenomena-specific patterns of interactions described by laws specific to those disciplines, often in mathematical form.
- What about equivalent impactful phenomena, theory, and mathematics for systems in general?
- We assert: More attention should be given to already modeled phenomena (from Hamilton and other STEM pioneers) before spending too much effort looking elsewhere. [25-26]



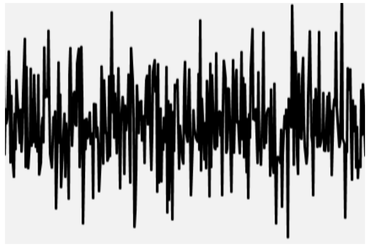
Informally, the Hamiltonian

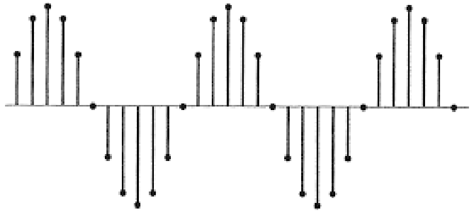


- Hamilton described an energy-like "characteristic function" of a system's state in a general and mathematical way <u>not restricted</u> to only mechanical or other "hard" engineered systems. [27]
- <u>Systems</u>: Start with a system of any type. By "system", we mean a set of interacting system components.
- <u>Interactions</u>: By "interact" we mean they exchange input-outputs, such as force, material, energy, or information, resulting in changes of state of the components.
- <u>States</u>: By "state" of a component we mean the condition of the component that can modify its current input-output behavior. Interaction thus changes state, which in turn impacts interaction.

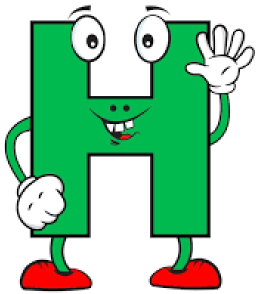


Non-deterministic and discrete systems





- This short and informal discussion focuses on deterministic, smoothly continuous systems, to build intuition.
- However, discrete Hamiltonian systems have been heavily explored and exploited, including providing the symplectic Hamiltonian integrators found in numerical simulation. [28-29]
- Non-deterministic cases: Hamiltonian mechanics also provide the foundations of the rich historical field of statistical mechanics, where state flows are replaced by probability density flows.[30-31]
- Probabilistic cases also re-enter this story through machine learning and human behavior.

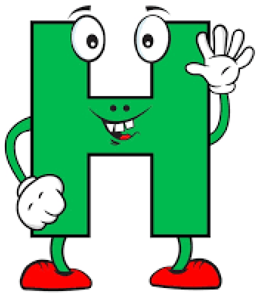


State trajectories; Hamilton's generalized momentum

- States: Have a way of representing the state of the system of interest $Q(t) = \{q_1,\ldots, q_n\}$, whose values change over time at rate $\dot{Q}(t) = \{\dot{q}_1, \ . \ . \ . \ , \dot{q}_n\}$, believed sufficient to characterize observed interactions.
- Characterizing System Level Behavior: Imagine now a scalar-valued function of state and time, *not yet defined here*, from Hamilton: ***H***$(Q, \dot{Q}, t)$, later $H(Q, P, t)$, intended to characterize something about the system--*we have not said how yet*.
- Momentum: Hamilton invented "generalized momentum", $P(t) = \{p_1,\ldots,p_n\}$, generalizing the idea of momentum in elementary physics--describing ability to change $\dot{Q}$. His generalized P is defined as sensitivity of H (*not defined here just yet*) to $\dot{Q}(t)$:

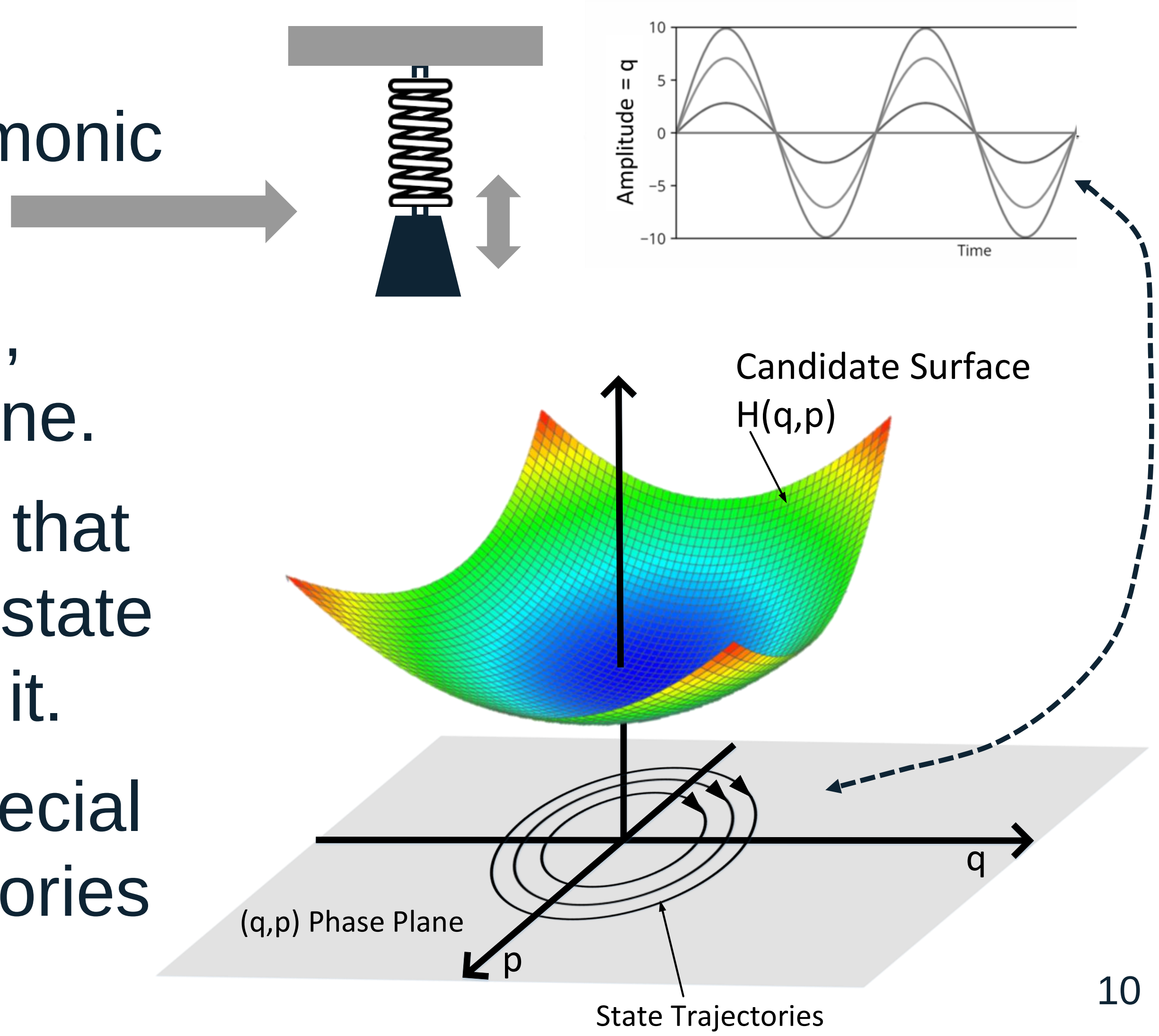
$$p_i \equiv \frac{\partial H}{\partial \dot{q}_i} \qquad \text{(Hamilton)} \qquad \text{Equation (1)}$$

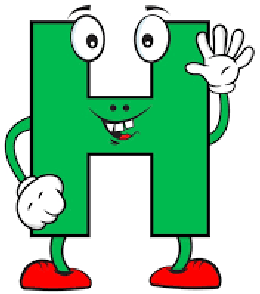
Intuition: If H turns out to be something "like" energy, this says momentum is sensitivity of energy to changes in velocity--or that energy is exchanged to change velocity.



Defining a surface, H(Q,P), that "characterizes" the family of trajectories

- Consider an undamped (simple harmonic oscillator) mass-spring system.
- We are looking for a smooth surface, H(Q, P) that sits above the (q, p) plane.
- Hamilton found a surface function H that "characterizes" the family of system state trajectories in the (q, p) plane below it.
- The Hamiltonian function has the special nature that "describes" all the trajectories below it! What could that be? . . . . .





Defining the Hamiltonian ---- H(Q,P): A different reasoning path to build intuition

- We want H to "characterize" (describe) the system's (Q,P) trajectories, and will do so by *tying those trajectories to the* *<u>local slopes of surface</u>* H(Q,P).

- This provides an <u>intuitive</u> way to define H(Q,P)….

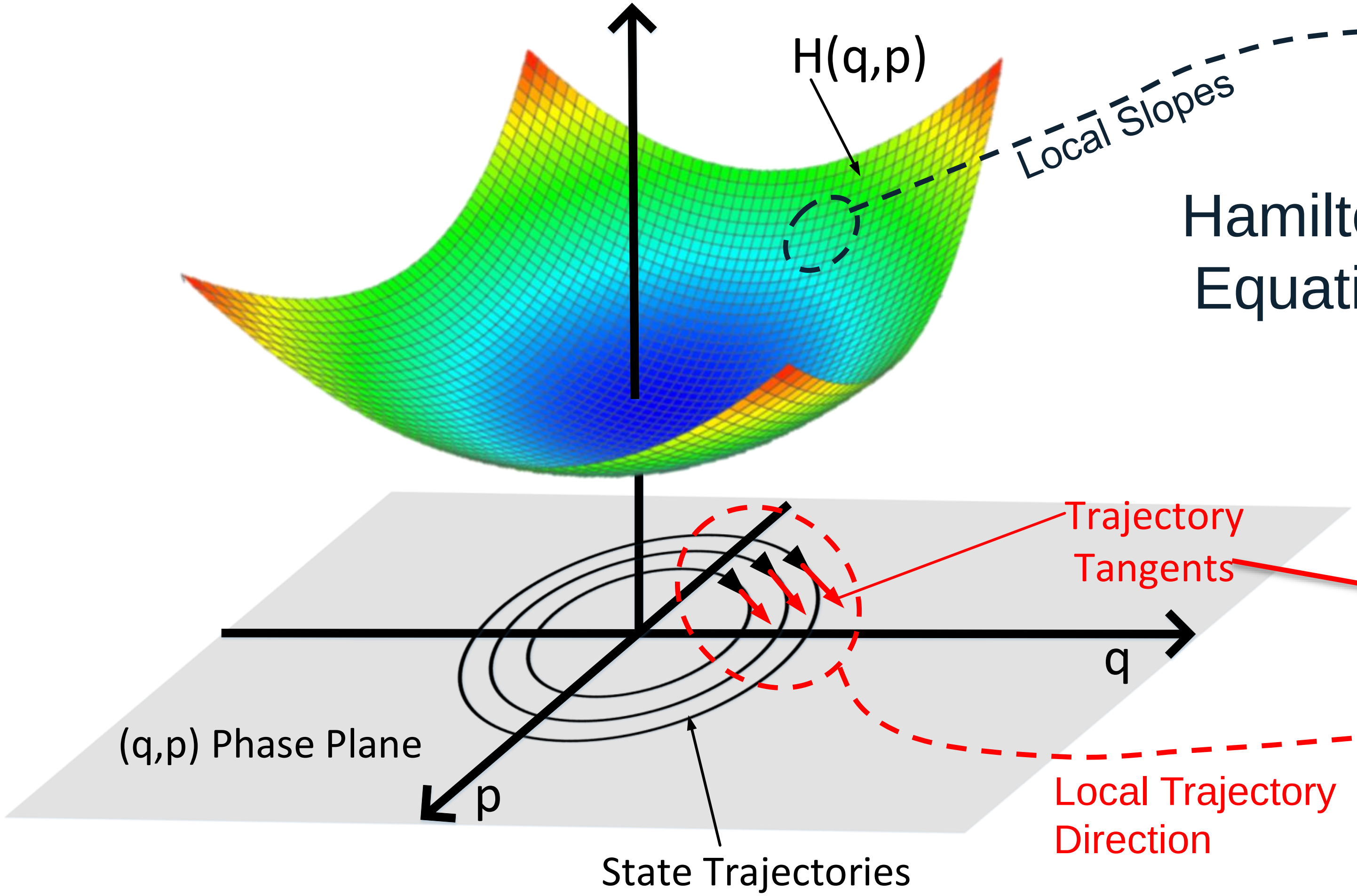
- First, pick the local sensitivity of H with respect to p_i at (q_i, p_i) is to be equal to the time rate of change of q_i along the system's state trajectory passing through (q_i, p_i):

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \text{(Hamilton)} \qquad \text{Equation (2)}$$

- Second, pick the local sensitivity of H with respect to q_i at (Q,P) is to be the negative of the time rate of change of p_i along the system's state trajectory passing through (q_i, p_i):

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad \text{(Hamilton)} \qquad \text{Equation (3)}$$

- Why does it work? . . . .



Hamilton's Equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \quad \text{Eqn (2)}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \quad \text{Eqn (3)}$$

Trajectory Tangent Vector =

$$\{\, \dot{q}_i \,,\, \dot{p}_i \,\} = \left\{ \frac{\partial H}{\partial p_i} \,,\, -\frac{\partial H}{\partial q_i} \right\}$$

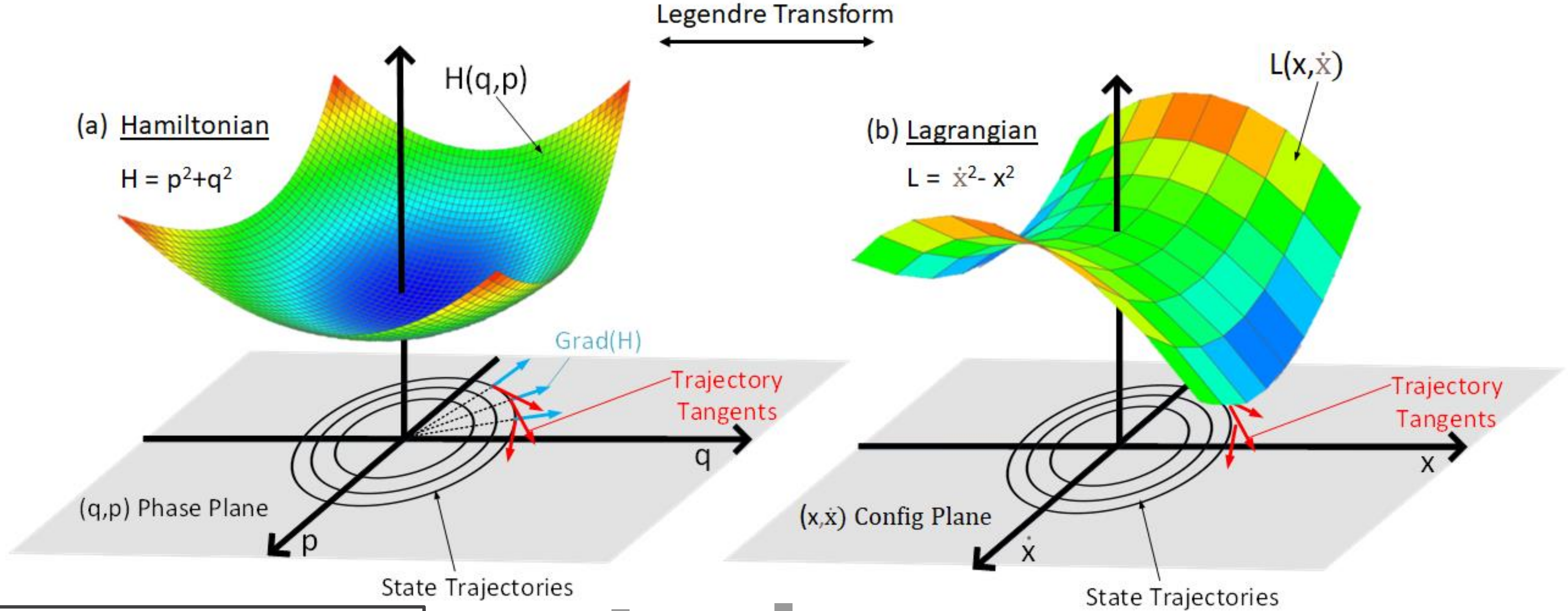
- For intuition, notice that dividing both sides of Eq (2) by both sides of Eq (3) shows that the instantaneous direction of motion in the (q, p) plane is the same as the ratio of the local surface slopes of H in the q and p directions.
- The relative slopes of H "steer" the trajectories in the (q, p) plane!
- So, the *surface* H(Q, P) "characterizes" all the dynamic *trajectories* in the (Q, P) plane.

Defining the Hamiltonian: A non-traditional path builds intuition and expands awareness of energies.

- The Traditional Sequence (based on recognized energies of familiar types):
  - Start from an accepted Lagrangian for a familiar system class, energies (e.g., mechanical).
  - Perform Legendre (purely mathematical) transformation to obtain Hamiltonian (H). [32-33]
  - H satisfies Hamilton's equations of motion, including generalized momentum, conservation of energy, etc., and has the advantage of being directly integrable via symplectic integrators.
- The Alternate Sequence (based on observation of state trajectories):
  - Start with any deterministic[1] system and its state variables (state 'positions', velocities).
  - Observe the state trajectories of the system over time.
  - Generate a "characteristic function" H *from the observed state trajectories*[2].
  - This H likewise satisfies Hamilton's equations of motion, defines a generalized momentum, and is integrable via symplectic integrators.
  - Provides a broader interpretation of P.E. and K.E. beyond just familiar mechanical and other "traditional" systems—energy as a "characteristic function" in spirit of Hamilton.

(1) Also formulated for probabilistic and discrete systems. (2) One interesting method: Machine learning.

Historically, the Lagrangian came before the Hamiltonian



$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad -\dot{p}_i = \frac{\partial H}{\partial q_i} \qquad \textit{where } p_i \equiv \frac{\partial H}{\partial \dot{x}_i}$$

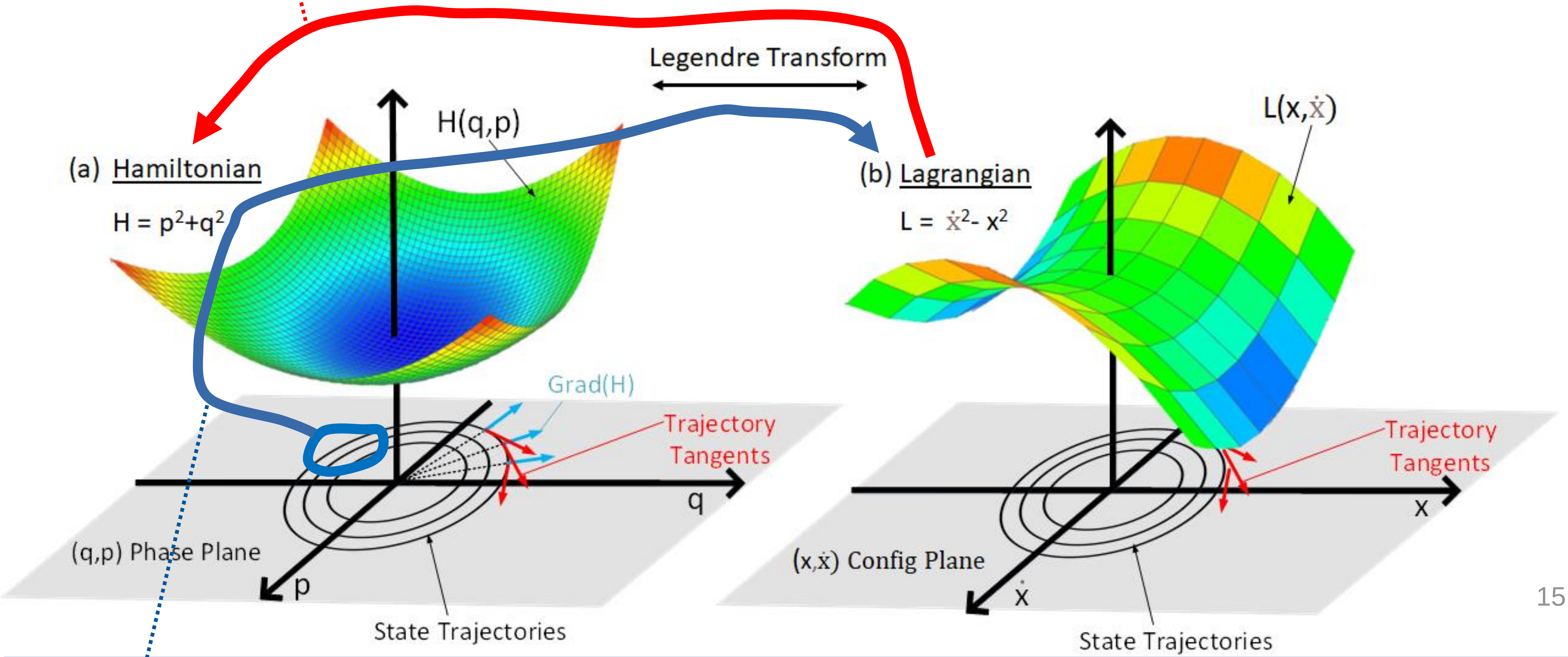
← Hamilton (first order)

Euler-Lagrange → (second order)

$$\frac{\partial L}{\partial x_i} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = 0$$

Traditional Reasoning Sequence: Assumes energy concept and leads to Hamiltonian [32-33]

The historical path assumed we already had a Lagrangian and familiar mechanical or other energies. The alternate path does not require those, and teaches us about “energies” of different systems.



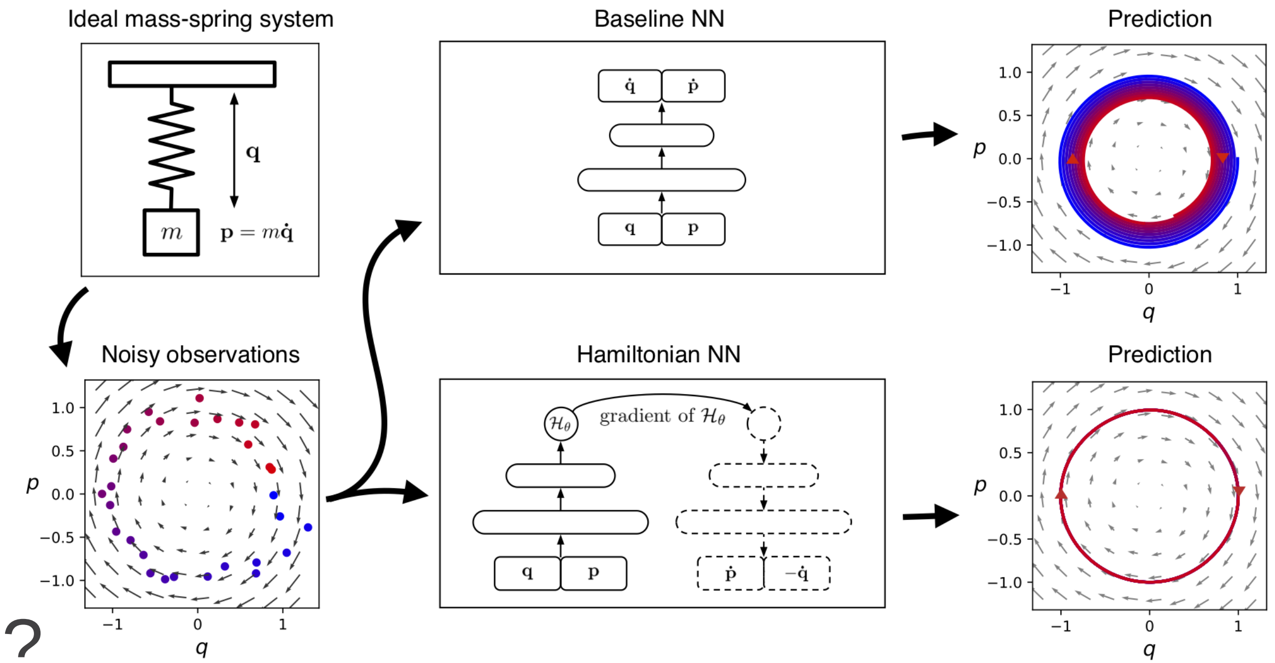
Alternate Reasoning Sequence: Assumes only States, Trajectories; leads directly to Hamiltonian and “energy”. Examining Grad(H) leads to invariance (conservation) of H along trajectories; variational/Lagrangian insight. [40]

Machine learning--yet another path to Hamiltonians: Physics-informed neural networks (PINNs)

- Hamiltonian Neural Networks (HNNs) are part of the emerging family of Physics-Informed Neural Networks (PINNs). [34]
- HNNs learn a Hamiltonian by observing empirical trajectory data.

- Used to learn Hamiltonians of:
  - Ocean currents [36]
  - Three body problems [37]
  - Molecules [38]
  - Stars orbiting a galactic center [39]

- Why not also Systems of Innovation?



From [36]

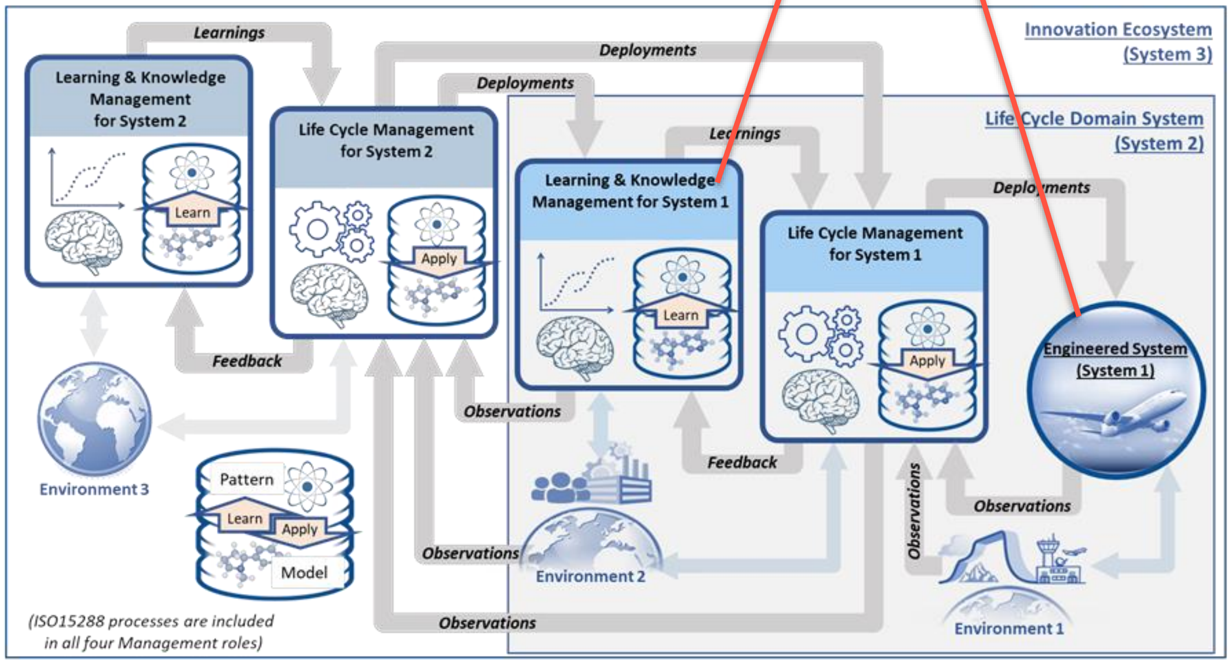
Application: States and learning in an innovation ecosystem

The diverse engineering, production, sustainment, other life cycle management processes of any innovation ecosystems can be characterized using the "Consistency Management" paradigm of INCOSE ASELCM Pattern: [22,42,43]

Application: June, 2023

Projects, seen as *learning:*

S2
S1



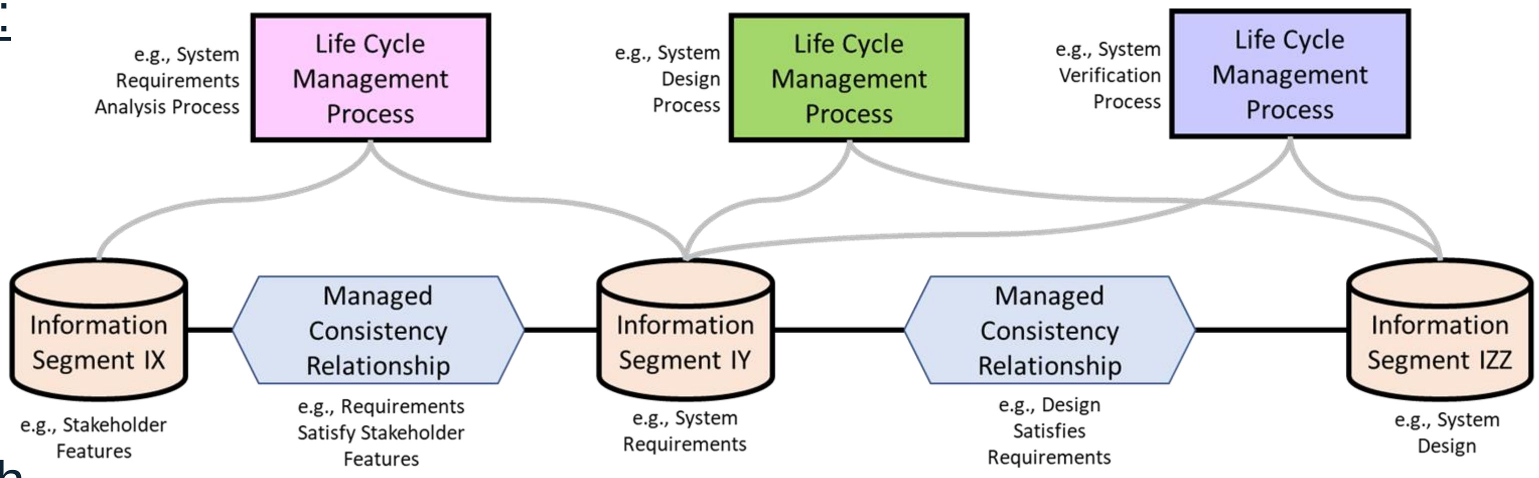
System 2 learns about **System 1** and its environment

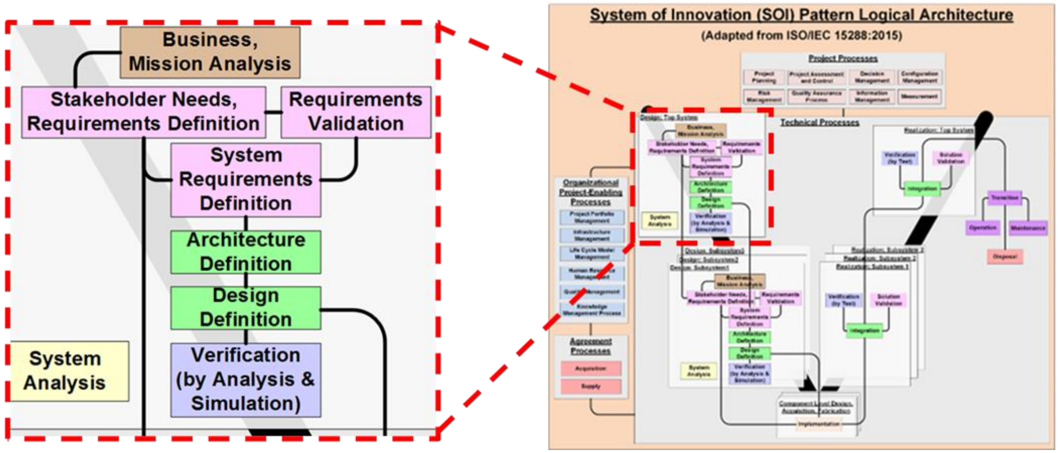
Application: Jan, 2023

The consistency management paradigm [43]

<u>Examples of managed "consistency pairs":</u>

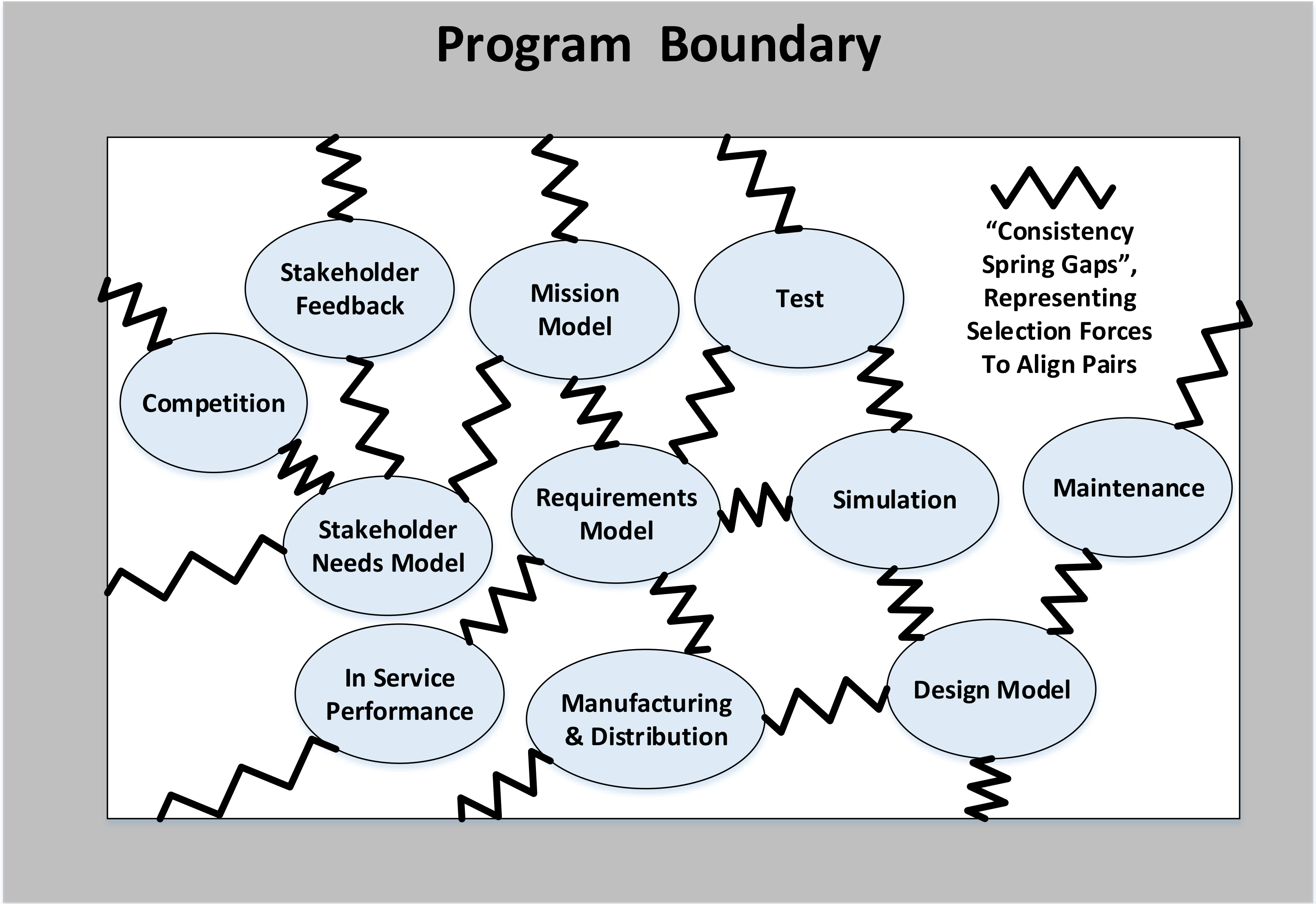
1. Design consistent with requirements?
2. Requirements consistent with mission and stakeholder needs and priorities?
3. Design consistent with experience?
4. Manufactured product consistent with design specifications?
5. Observed use of product consistent with the product mission and requirements?
6. Performance of deployed product consistent with specified requirements?
7. Environment of product use consistent with product mission and requirements?
8. Simulation outputs consistent with empirical observations?
9. Design consistent with good practice?
10. And many, many more . . .





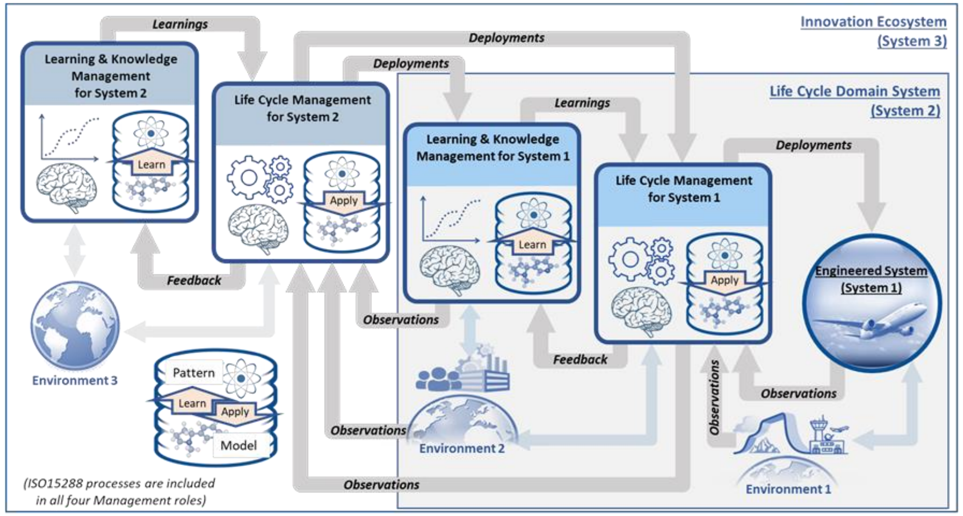
The consistency management paradigm

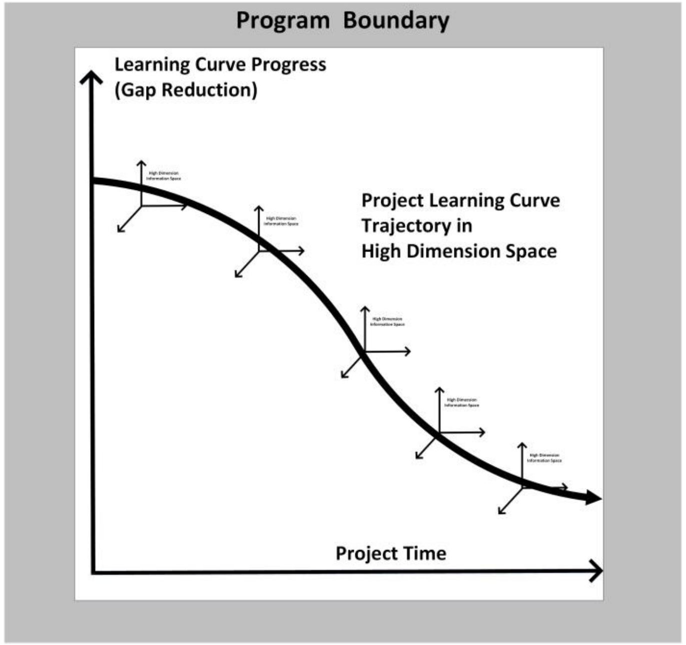
- **<u>During a project</u>**: Consistency gap “springs” jiggle until we reduce gaps to acceptable levels of consistency.
- A model of all innovation ecosystems—good ones as well as not so good ones.
- What are the dynamics of such systems?
- What are the selection “forces”?
- What is the “energy”?
- What is overall “trajectory” in this “state space”?

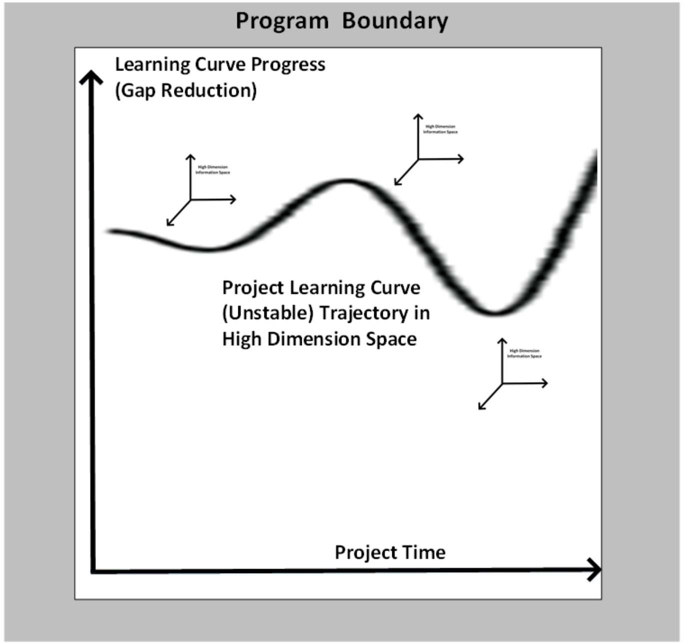


Overall trajectories, seen as:

- Boundary value problem
- Learning curve problem



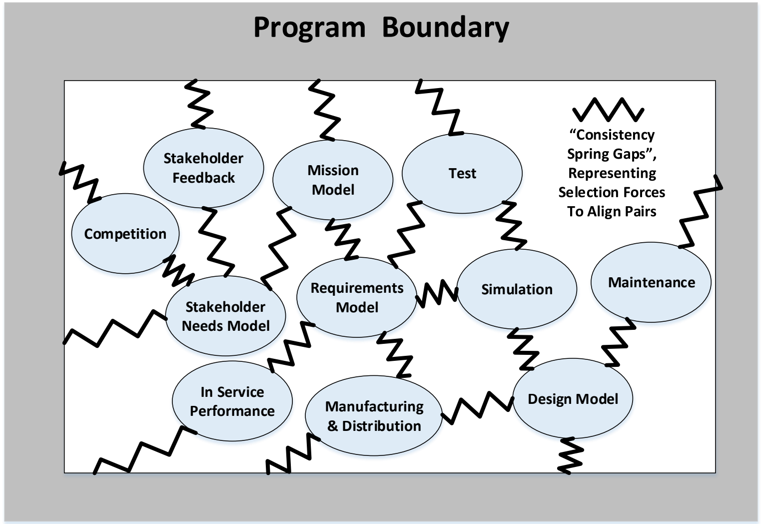


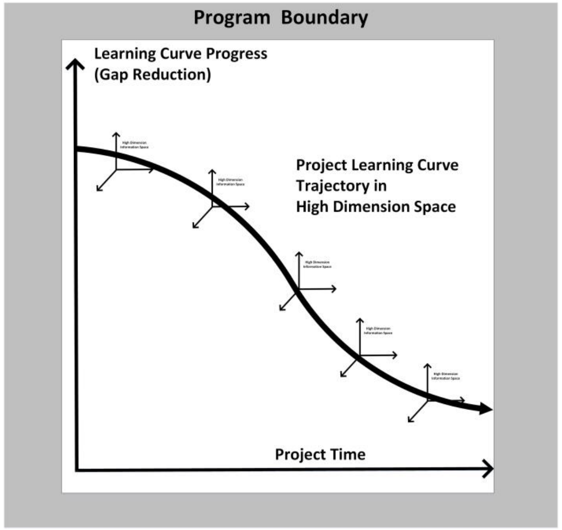


A well-behaved project reduces uncertainty, consistency gaps.

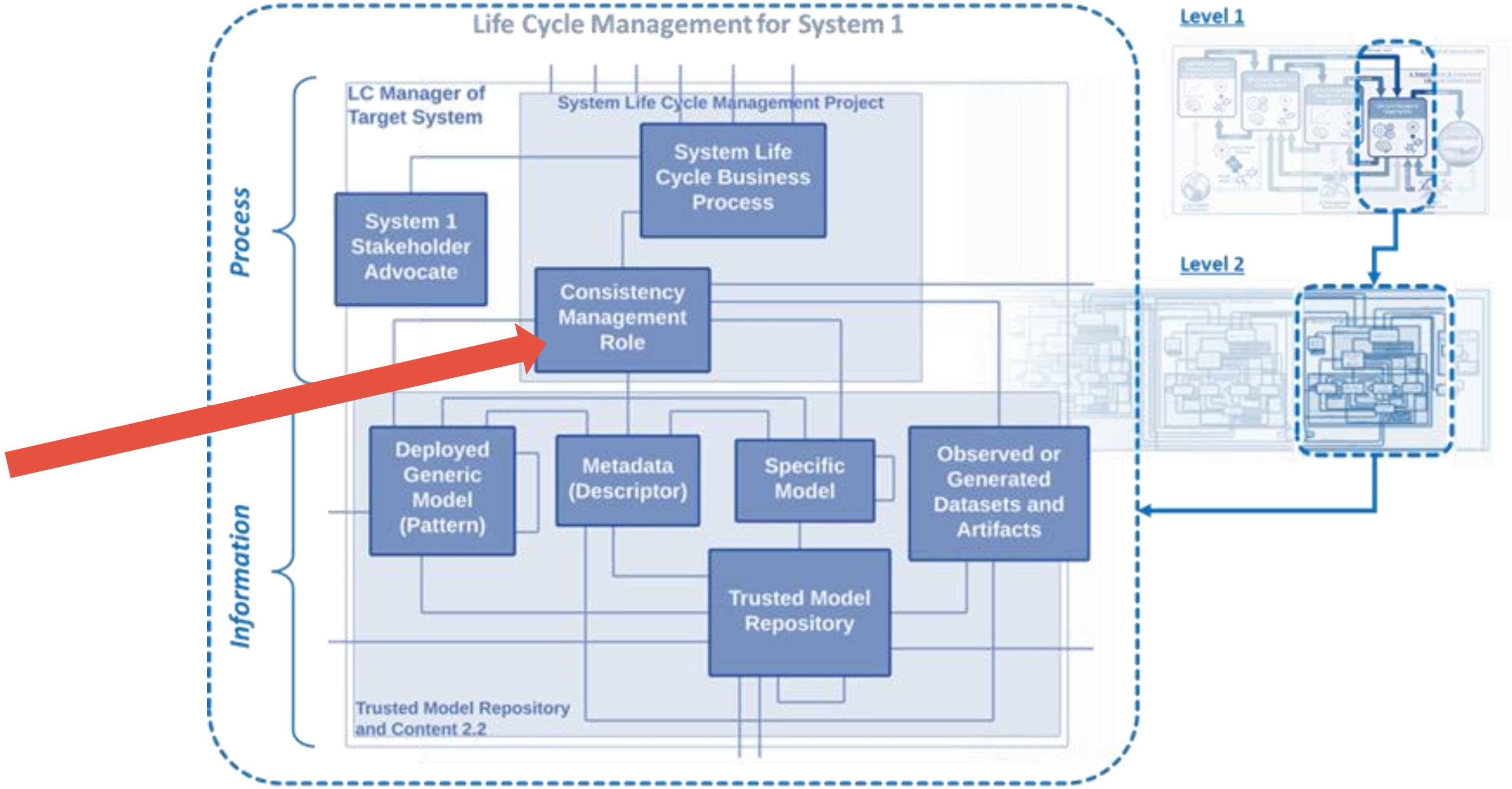
Other projects can be unstable.

<u>In Engineering Projects</u>: Machine learning, human learning, System 2 energy and momentum





- If current knowledge is viewed as state, can we use sizes of its "consistency gaps" as a form of "potential energy" for discovering Hamiltonians?
- Confidence in learned patterns, uncertainty management, Bayes [66]
- John Hopfield's seminal 1982 PNAS paper rekindled research in neural network learning machines, referring to "energy" functions "isomorphic to an Ising model" (physics). [48]
- Contemporary machine learning abounds in "energy-based methods". [49, 67]
- Recognizing the deep connections between performance of neural nets over large sample spaces and the probability distributions of statistical physics. [50]



- Executing real projects involves *<u>making decisions</u>*:
  - Incremental, minor but accumulating decisions;
  - Major milestone, stage-gate decisions.
- *<u>We can view all such decisions as reconciling inconsistencies</u>*. [54, 55]
- The ASELCM reference model of the digital thread represents the roles of detecting and reconciling inconsistencies, along with recording them. [41]

Classical mathematical physics models: Practical for socio-technical systems?

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = - \frac{\partial H}{\partial q_i}$$

- Are Hamilton’s mathematical models even plausible for describing complex, human-performed socio-technical systems, such as engineering or other innovation life cycle management?
- That such a question would even be seriously considered has recently become more likely, based on advancements in machine learning leading to surprising demonstrations. [18]

Ecosystem selection forces, dissipation, entropy, and complexity

- For a future discussion:
  - Selection forces [53-55]
  - Hamiltonian learning systems [56]
  - Innovation ecosystem dissipation and reversibility [57]
  - Entropy and its preservation in Hamiltonian systems [58]
  - The question of non-holonomic constraints [61-64]

So what? Conclusions and future work

This paper [65] has:

1. Outlined some of the questions faced by Innovation Ecosystem projects;
2. Provided an informal refresher on how Hamilton's framework applies to diverse systems, including socio-technical and information systems, and ASELCM Pattern;
3. Shown that the key Innovation Ecosystem state variables relevant for Hamiltonian "potential" modeling include Consistency Management "gaps" central to digital thread;
4. Noted Energy Based Learning methods for machine learning are already being used to learn real system Hamiltonians as well as being Hamiltonian modeled themselves;
5. Shown that consistency management's needs for inconsistency detection and reconciliation are candidates for machine learning based aids to labor-intense roles;
6. Shown that this synthesis suggests an Innovation Enterprise architecture integrating the digital thread as well as machine and human learning;
7. Laid a foundation for future momentum kinetics and applications work utilizing these approaches, as well as case study work.

Questions, discussion

Thank you!

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