International Council on Systems Engineering
A better world through a systems approach

Bifurcation Analysis for System Resilience: A Case Study on Power Infrastructure

Rogelio Gracia Otalvaro & Bryan C. Watson
Embry-Riddle Aeronautical University

INCOSE International Symposium 2025 | Ottawa, Canada

Presentation Overview

- Resilience as a defense against fragility
- Framework: 6-step Bifurcation Analysis / Resilience method
- Case Study: IEEE 9-Bus analysis
- Resilience Study: absorptive, adaptive, recovery capacities
- Conclusion

Presentation Overview

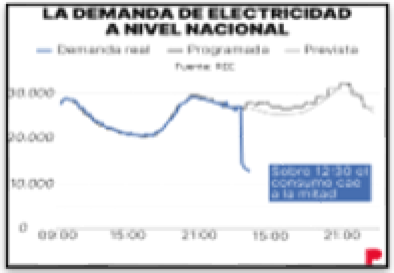
- Resilience as a defense against fragility
- Framework: 6-step BA / Resilience method
- Case Study: IEEE 9-Bus analysis
- Resilience Study: absorptive, adaptive, recovery capacities
- Conclusion

Modern systems are more fragile than ever

We build faster than we understand, leading to uncertainty and risks

London Heathrow, March 2025

CrowdStrike, July 2024



Spain, April 2025

UK, yesterday

INCOSE

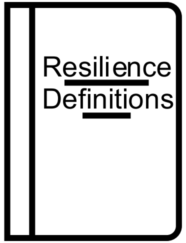
https://www.bloomberg.com/news/newsletters/2025-03-25/the-device-that-brought-down-heathrow
https://mashable.com/article/microsoft-crowdstrike-outage-millions-more-computers-affected
https://www.farodevigo.es/sociedad/2025/04/28/apagon-general-deja-luz-media-espana-116817439.html

Resilience Engineering is emerging within Systems Engineering to understand how to navigate disruptions

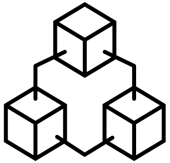
Absorb **Adapt** **Recover**

INCOSE

Resilience Engineering rises as a way to better understand systems under stress, but it comes with challenges



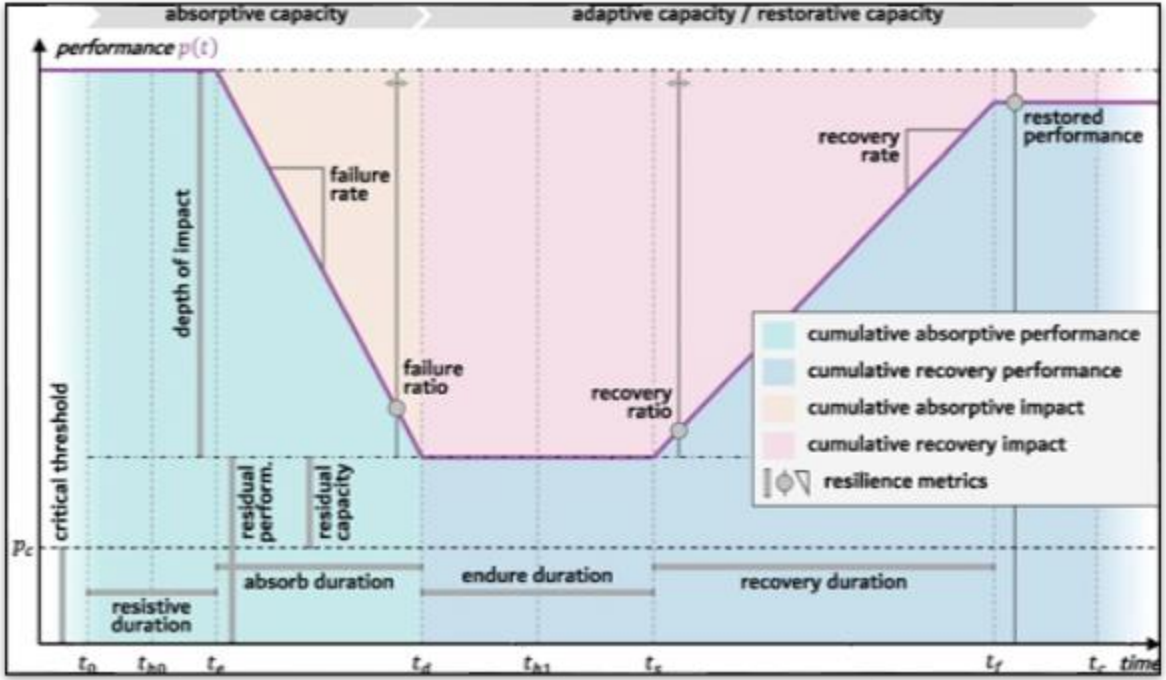
Lack of Standardization



Complex Dynamics and Integrations

Data Requirements

Current resilience methods do not always give insight into why disruptions happen

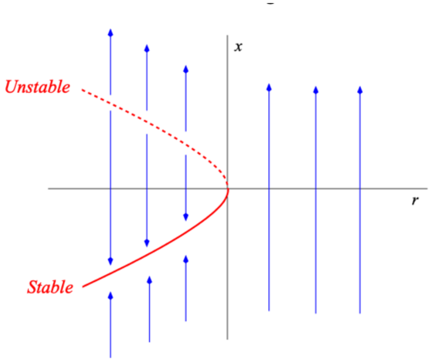


(Bruckler, et al., 2024)

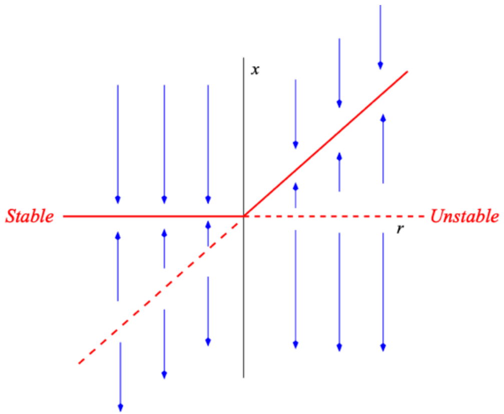
Presentation Overview

- Introduction
- Framework: 6-step Bifurcation Analysis / Resilience method
- Case Study: IEEE 9-Bus analysis
- Resilience Study: absorptive, adaptive, recovery capacities
- Conclusion

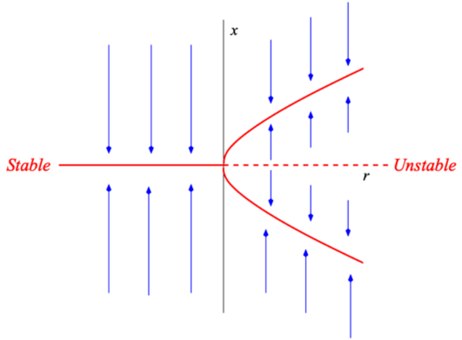
Bifurcation Analysis, from Dynamic Systems Theory, studies how small parameter variations can impact system stability



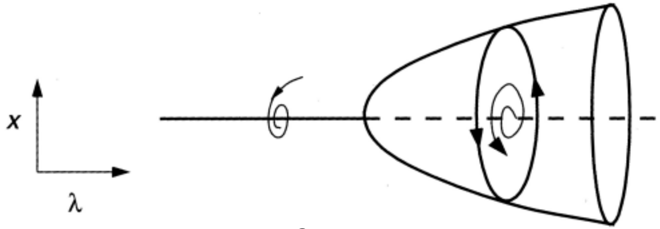
Saddle-Node



Transcritical

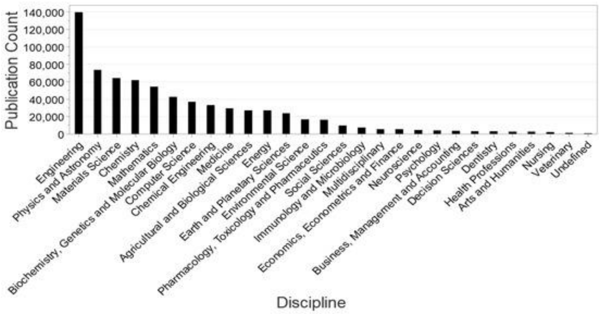


Pitchfork

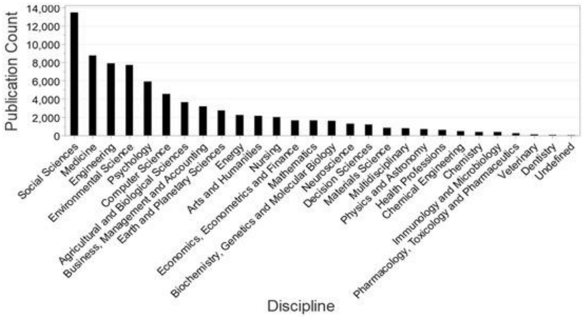


Hopf

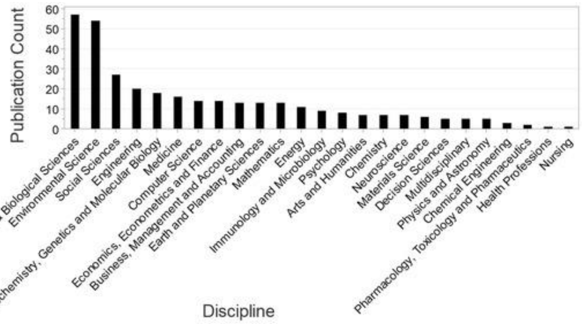
Bifurcation Analysis is not common in Resilience Engineering



Publication counts listing stability in their titles.



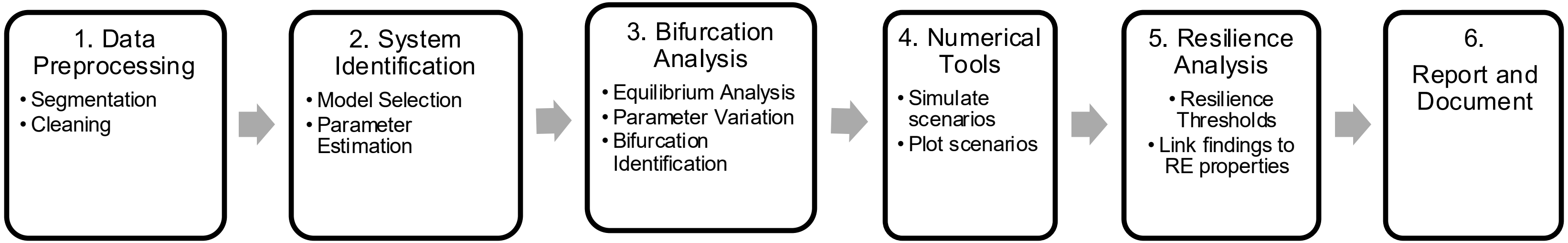
Publication counts listing resilience in their titles.



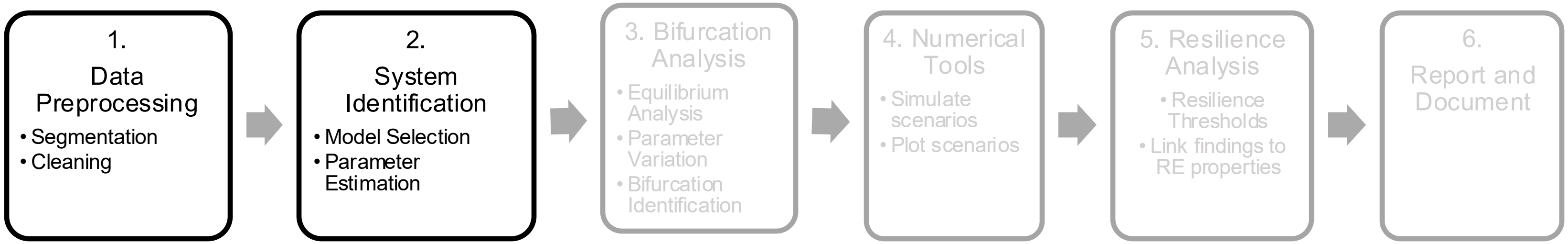
Publication counts listing both stability and resilience in their titles.

(Mayar, Carmichael, & Shen, 2022).

6-Step Framework to use BA in RE



To analyze a system, a system model is required



Bifurcation Analysis benefits from the use of numerical tools

3. Bifurcation Analysis
- Equilibrium Analysis
- Parameter Variation
- Bifurcation Identification

→

4. Numerical Tools
- Simulate scenarios
- Plot scenarios

→

5. Resilience Analysis
- Resilience Thresholds
- Link findings to RE properties

→

6. Report and Document

$$x_{n+1} = r * x_n * (1 - x_n)$$

$$x_o = r * x_o * (1 - x_o)$$

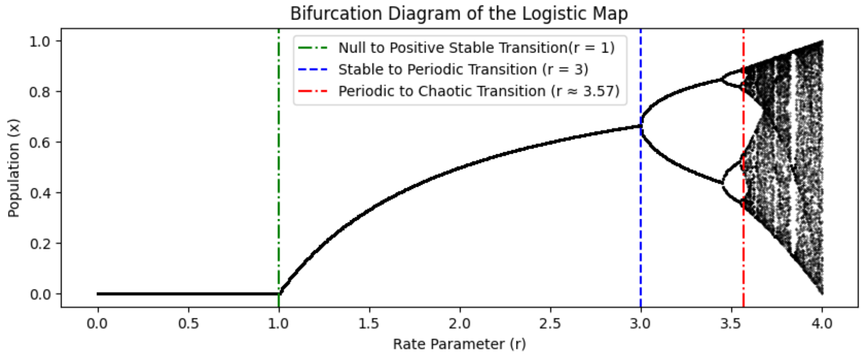
$$x_o = r * x_o - r * (x_o)^2$$

$$0 = r * (x_o)^2 - (r - 1) * x_o$$

$$\frac{d}{dx_n}(r * x_n * (1 - x_n)) = r - 2 * r * x_n$$

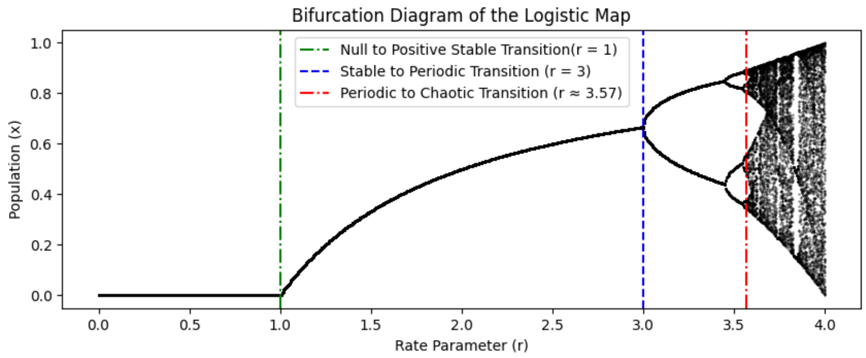
$$\left.\frac{dx_{n+1}}{dx_n}\right|_{\frac{r-1}{r}} = r - 2 * r * \frac{r-1}{r} = 2 - r = 1$$

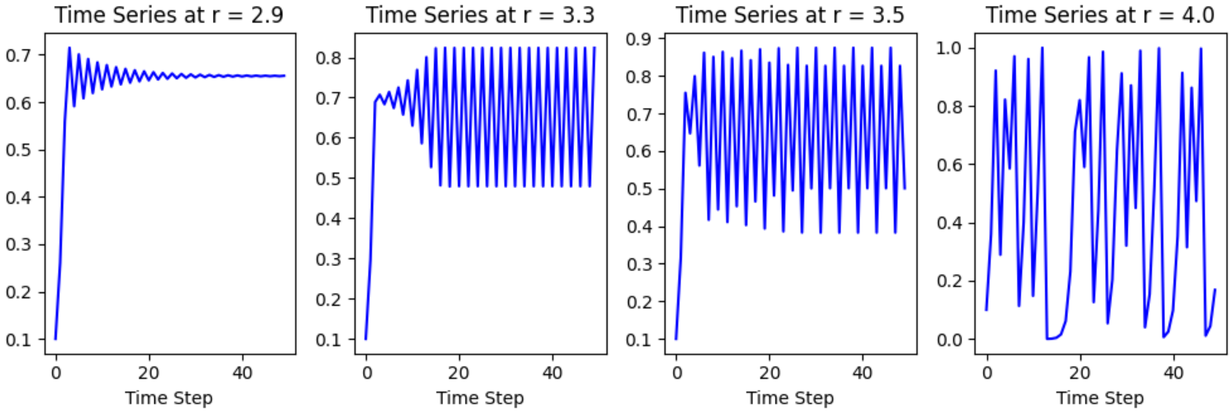
$$\begin{cases} r = 1 \\ r = 3 \end{cases}$$



INCOSE

Bifurcation Analysis benefits from the use of numerical tools





Linking BA insights with Resilience Capabilities

5. Resilience Analysis
- Resilience Thresholds
- Link findings to RE properties

→

6. Report and Document

		Bifurcation Analysis		
		Time Series Analysis	**Phase Diagram**	**Bifurcation Diagram**
Resilience Capabilities	**Absorptive Capabilities**	High absorptive capacity is indicated if the system experiences a disturbance (visible as a deviation from a steady state) but quickly returns to its baseline or changes minimally.	A system with high absorptive capacity might show trajectories in the phase diagram that quickly return to a stable orbit or fixed point after a disturbance.	A system with high absorptive capacity would demonstrate similar qualitative behavior over a wide range of parameter values.
	Adaptive Capabilities	A system that presents different states and can smoothly transition from one to another shows high adaptability.	A system transitioning smoothly between different orbits or fixed points as conditions change suggests high adaptability, as it can function across a range of dynamic states.	A system is adaptable if it can transition smoothly between different types of behavior (e.g., from stable to periodical states) as parameters change, indicating flexibility.
	Recovery Capabilities	If the system settles into a new pattern or fails to stabilize after a disturbance, it suggests limited recovery capability.	If paths leading back to a stable state or fixed point following a perturbation, it indicates good recovery properties. Conversely, if the system moves to a different attractor or becomes chaotic, it suggests limited recovery.	Good recovery will be shown if after passing through a bifurcation point and then reversing the parameter change, the system returns to its original behavior. It this is not the case, there could be hysteresis in the system, limiting recovery.

(Gracia Otalvaro & Watson, 2024)

Presentation Overview

- Introduction
- Framework: 6-step BA / Resilience method
- Case Study: IEEE 9-Bus analysis
- Resilience Study: absorptive, adaptive, recovery capacities
- Conclusion

IEEE or WSCC 9 Bus is evaluated from 10% to 230% of its nominal load

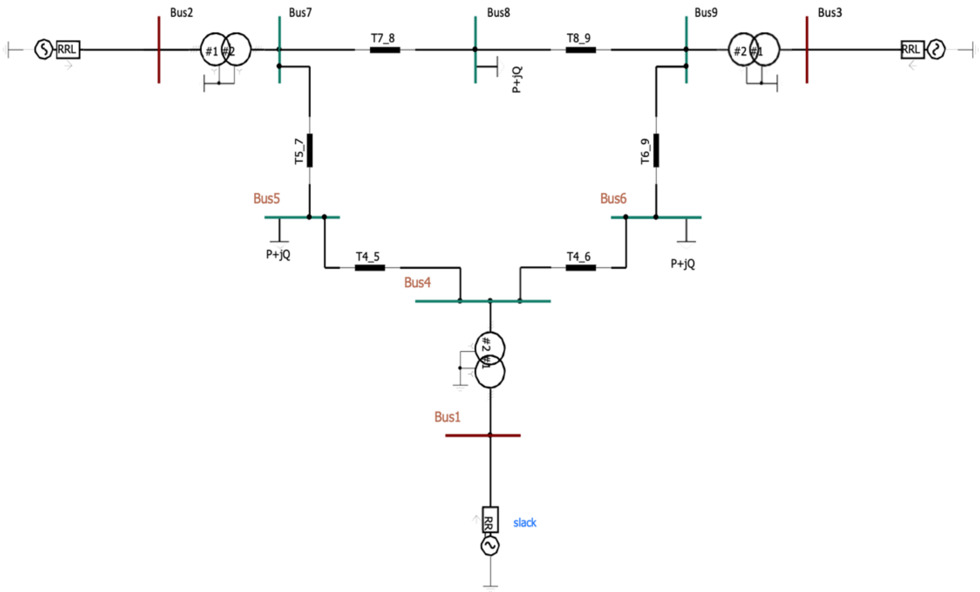


Figure 1: Model of the IEEE 9 Bus System (PSCAD, 2018)

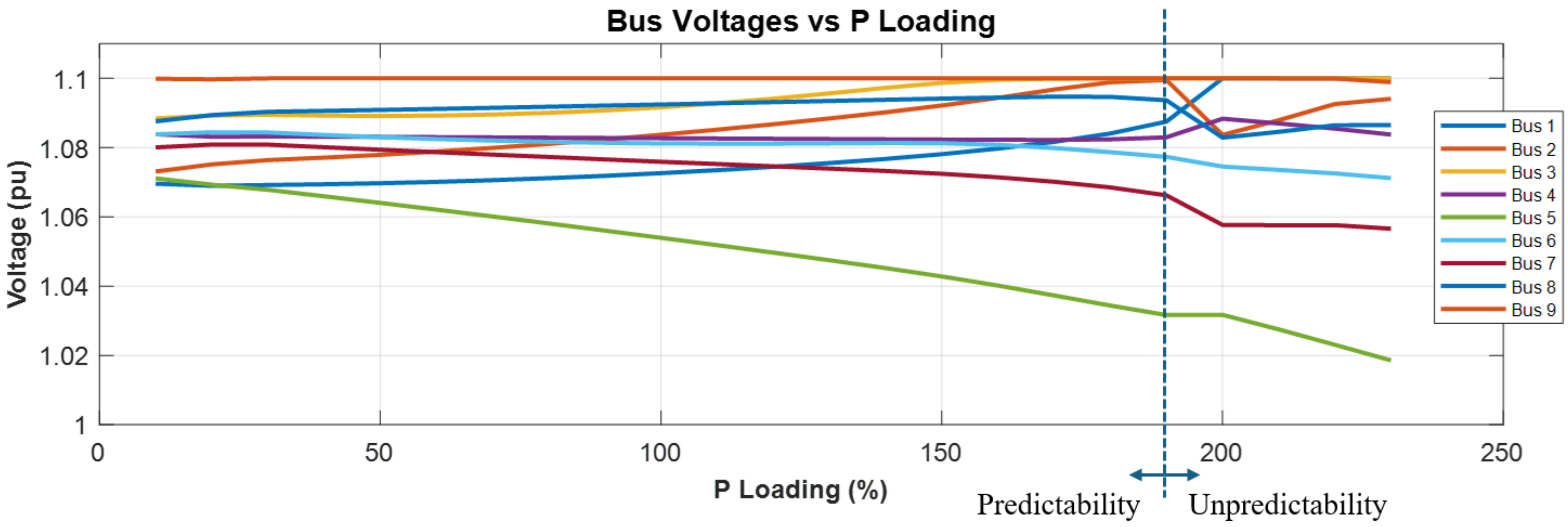
Feature	Value
Number of buses	9
Generators	3 (located at buses 1, 2, and 3)
Loads	**3 (at buses 5, 6, and 8) with nominal values at 315MW**
Transformers	3 (with off-nominal tap ratios)
Transmission lines	9 (forming a meshed network)

$$State\ Vector = [V_i, \theta_i]$$

$$Active\ Power\ MW \quad P_{e,i} = V_i \sum_j V_j Y_{ij} \cos(\theta_i - \theta_j - \angle Y_{ij})$$

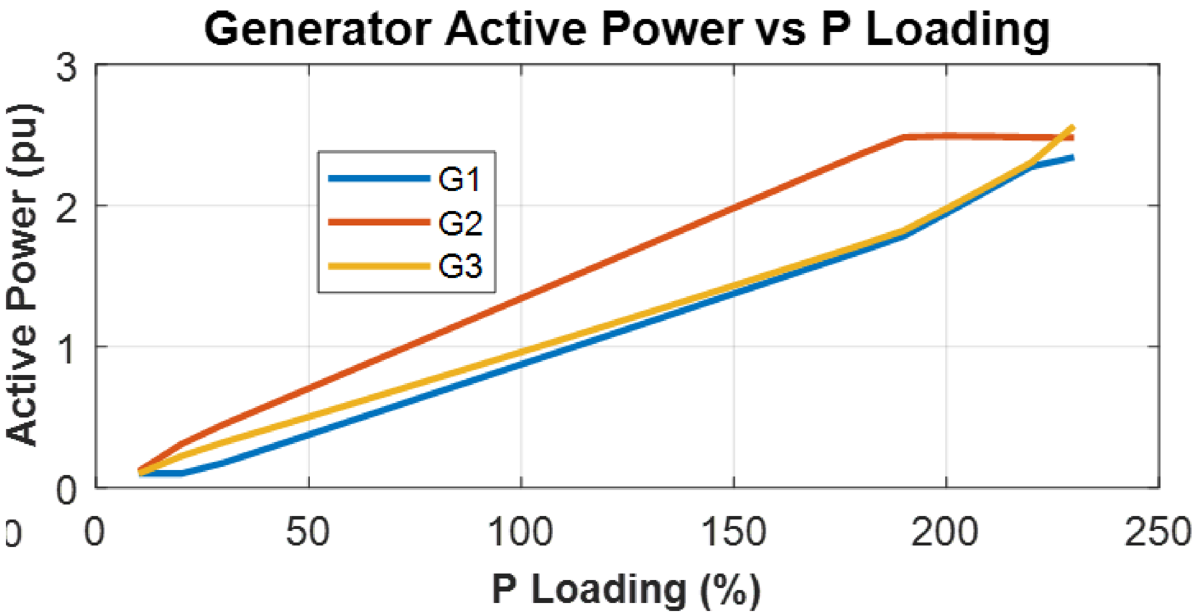
$$Reactive\ Power\ MVar \quad Q_{e,i} = V_i \sum_j V_j Y_{ij} \sin(\theta_i - \theta_j - \angle Y_{ij})$$

Voltages remain predictable until 190% of nominal consumption, where behavior changes



INCOSE

Generator output evaluation shows the reason for change in behavior (G2)



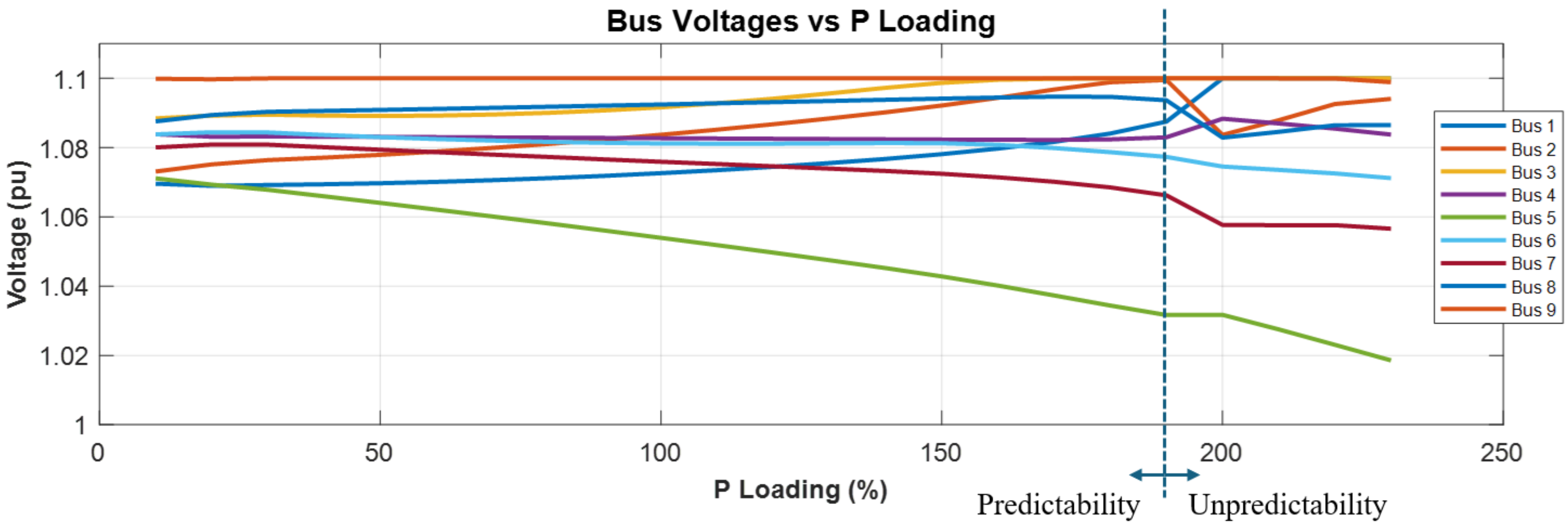
Presentation Overview

- Introduction
- Framework: 6-step BA / Resilience method
- Case Study: IEEE 9-Bus analysis
- Resilience Study: absorptive, adaptive, recovery capacities
- Conclusion

		Bifurcation Analysis		
		Time Series Analysis	**Phase Diagram**	**Bifurcation Diagram**
Resilience Capabilities	**Absorptive Capabilities**	High absorptive capacity is indicated if the system experiences a disturbance (visible as a deviation from a steady state) but quickly returns to its baseline or changes minimally.	A system with high absorptive capacity might show trajectories in the phase diagram that quickly return to a stable orbit or fixed point after a disturbance.	A system with high absorptive capacity would demonstrate similar qualitative behavior over a wide range of parameter values.
	Adaptive Capabilities	A system that presents different states and can smoothly transition from one to another shows high adaptability.	A system transitioning smoothly between different orbits or fixed points as conditions change suggests high adaptability, as it can function across a range of dynamic states.	A system is adaptable if it can transition smoothly between different types of behavior (e.g., from stable to periodical states) as parameters change, indicating flexibility.
	Recovery Capabilities	If the system settles into a new pattern or fails to stabilize after a disturbance, it suggests limited recovery capability.	If paths leading back to a stable state or fixed point following a perturbation, it indicates good recovery properties. Conversely, if the system moves to a different attractor or becomes chaotic, it suggests limited recovery.	Good recovery will be shown if after passing through a bifurcation point and then reversing the parameter change, the system returns to its original behavior. It this is not the case, there could be hysteresis in the system, limiting recovery.

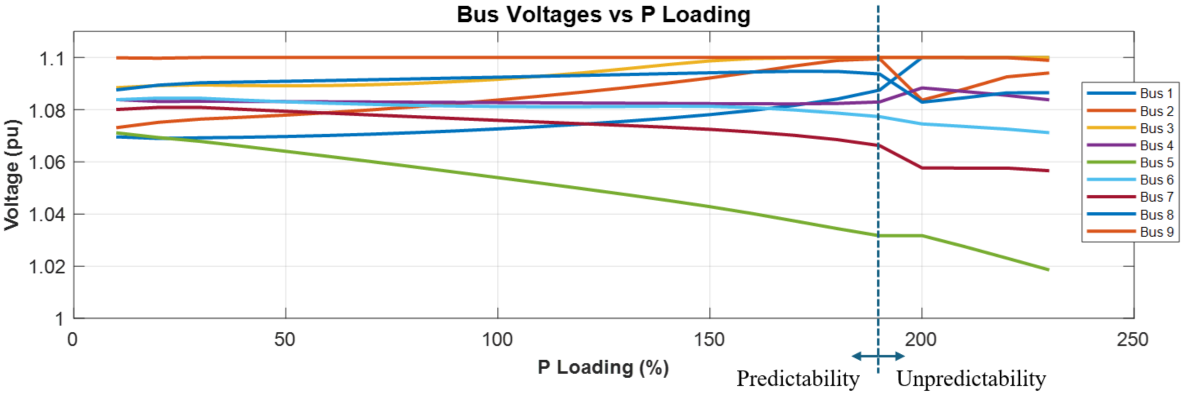
INCOSE

Absorptive Capabilities: how much change can the system handle before it changes its behavior?

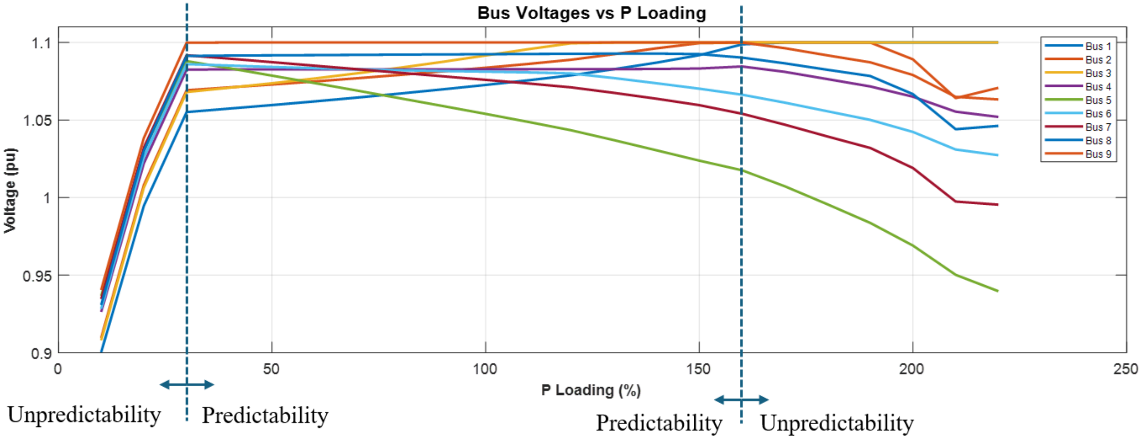


INCOSE

Adaptive Capabilities: How large is the system's operating range?

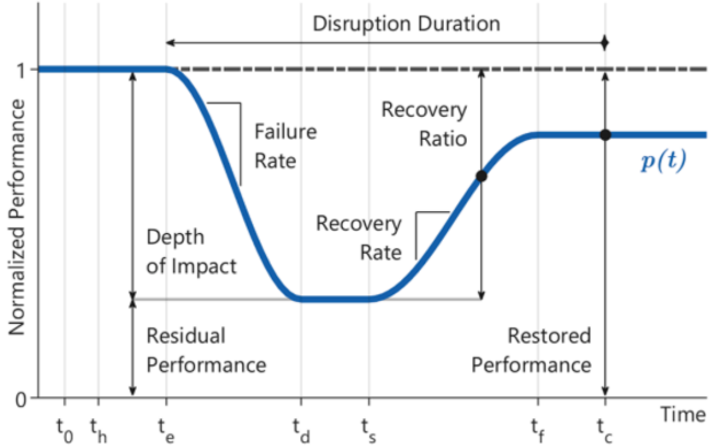


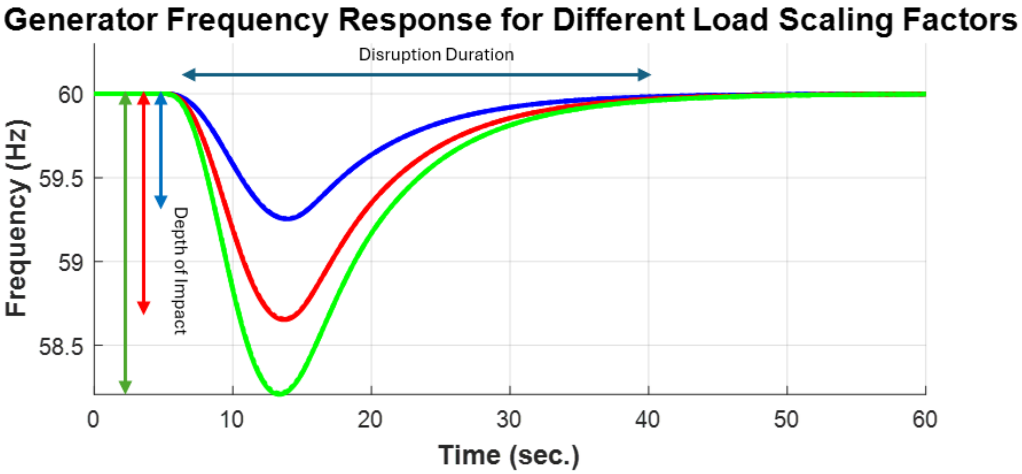
Only active load is increased



Both active and reactive loads are increased

Recovery Capabilities: How does the system react to events?





INCOSE

Presentation Overview

- Introduction
- Framework: 6-step Bifurcation Analysis / Resilience method
- Case Study: IEEE 9-Bus analysis
- Resilience Study: absorptive, adaptive, recovery capacities
- Conclusion

Conclusion

- **Resilience Engineering is emerging within Systems Engineering to understand how to navigate disruptions**
- **Studying a system's absorptive, adaptive, and recovery capabilities leads to resilient systems**
- **Analysis tools must keep up with the complexities seen in modern systems**
- **Bifurcation Analysis helps SE by identifying stability thresholds and providing resilience insights**

Next steps: Expanding the framework by adding more methods to it and testing it in more types of systems

		Bifurcation Analysis		
		Time Series Analysis	**Phase Diagram**	**Bifurcation Diagram**
Resilience Capabilities	Absorptive Capabilities	High absorptive capacity is indicated if the system experiences a disturbance (visible as a deviation from a steady state) but quickly returns to its baseline or changes minimally.	A system with high absorptive capacity might show trajectories in the phase diagram that quickly return to a stable orbit or fixed point after a disturbance.	A system with high absorptive capacity would demonstrate similar qualitative behavior over a wide range of parameter values.
	Adaptive Capabilities	A system that presents different states and can smoothly transition from one to another shows high adaptability.	A system transitioning smoothly between different orbits or fixed points as conditions change suggests high adaptability, as it can function across a range of dynamic states.	A system is adaptable if it can transition smoothly between different types of behavior (e.g., from stable to periodical states) as parameters change, indicating flexibility.
	Recovery Capabilities	If the system settles into a new pattern or fails to stabilize after a disturbance, it suggests limited recovery capability.	If paths leading back to a stable state or fixed point following a perturbation, it indicates good recovery properties. Conversely, if the system moves to a different attractor or becomes chaotic, it suggests limited recovery.	Good recovery will be shown if after passing through a bifurcation point and then reversing the parameter change, the system returns to its original behavior. It this is not the case, there could be hysteresis in the system, limiting recovery.

International Council on Systems Engineering
A better world through a systems approach

Bifurcation Analysis for System Resilience: A Case Study on Power Infrastructure

Rogelio Gracia Otalvaro & Bryan C. Watson
Embry-Riddle Aeronautical University

INCOSE International Symposium 2025 | Ottawa, Canada

INCOSE

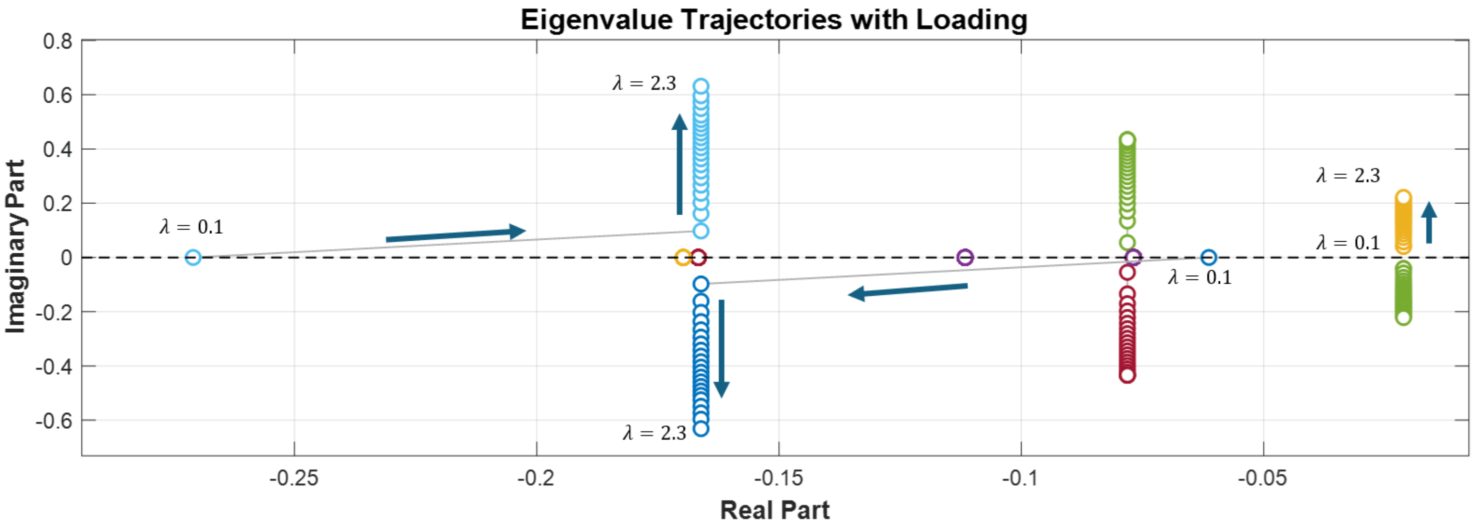
35th Annual INCOSE international symposium

hybrid event

Ottawa, Canada
July 26 - 31, 2025

While the system remains stable, oscillatory behavior arises as load increases

Eigenvalues show if the system is stable or unstable



Bifurcation Analysis studies how small parameter variations can impact system stability

$$f_1(X,Y,Z,t,p) = \frac{dX}{dt}$$

$$f_2(X,Y,Z,t,p) = \frac{dY}{dt}$$

$$f_3(X,Y,Z,t,p) = \frac{dZ}{dt}$$

Define System

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial X} & \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial Z} \\ \frac{\partial f_2}{\partial X} & \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial Z} \\ \frac{\partial f_3}{\partial X} & \frac{\partial f_3}{\partial Y} & \frac{\partial f_3}{\partial Z} \end{bmatrix}$$

Compute Jacobian

$$\det(J - \lambda I) = 0$$

If $\mathrm{Re}(\lambda) < 0$, stable

Find Eigenvalues $\lambda(p)$

INCOSE

1 Classical (second-order) "swing" model

Suitable when stator transients are neglected and the machine is represented by a constant internal emf **E′** behind the transient reactance **X′**.

$$\dot{\delta} = \omega_b \, (\omega - 1)$$

$$M \, \dot{\omega} = P_m - P_e - D(\omega - 1)$$

Symbol	Meaning
δ	rotor electrical angle (rad)
ω	rotor speed (p.u. of synchronous speed)
ω_b	base speed (rad s^{-1})
$M = 2H/\omega_b$	inertia constant (s) with H = kinetic energy stored at rated speed (MJ/MVA)
D	damping coefficient
P_m	mechanical power input
(P_e = \tfrac{	E'

2 Fourth-order (transient) d-q model

Adds the dominant field-winding and damper dynamics; widely used by grid codes and IEEE model libraries.

$$\begin{aligned}\dot{\delta} &= \omega_b\,(\omega - 1)\\ M\,\dot{\omega} &= P_m - P_e - D(\omega - 1)\\ T'_{do}\,\dot{E}'_q &= -E'_q - (X_d - X'_d)\,I_d + E_f\\ T'_{qo}\,\dot{E}'_d &= -E'_d + (X_q - X'_q)\,I_q\end{aligned}$$

$$P_e = E'_q I_q + E'_d I_d$$

$$I_d = \frac{V_d - E'_d}{X'_q}$$

$$I_q = \frac{V_q - E'_q}{X'_d}$$

Excitation- and governor-system add-ons

The electrical field voltage E_f is produced by an **Automatic Voltage Regulator (AVR)** and exciter, typically modeled as:

$$T_A \dot{E}_f = -E_f + K_A\,(V_{\text{ref}} - V_t)$$

Mechanical power P_m is regulated by a **turbine governor** (e.g., IEEE-G1, GGOV). Including these controllers closes the electromechanical loop for long-term dynamic simulations.

Parameter	Meaning
E'_q, E'_d	transient internal emfs (q- and d-axes)
T'_{do}, T'_{qo}	open-circuit transient time constants
X_d, X_q	synchronous reactances
X'_d, X'_q	transient reactances
V_d, V_q	stator terminal voltages in the rotor frame
E_f	field voltage (output of excitation system)

Bifurcation Examples

1. Saddle-Node Bifurcation

Equation:

$$\frac{dx}{dt} = r + x^2$$

Behavior:

- For $r < 0$: two fixed points (one stable, one unstable)
- For $r = 0$: one semi-stable fixed point (merging point)
- For $r > 0$: no fixed points

2. Pitchfork Bifurcation (supercritical)

Equation:

$$\frac{dx}{dt} = rx - x^3$$

Behavior:

- For $r < 0$: one stable fixed point at $x = 0$
- For $r > 0$: $x = 0$ becomes unstable, two new stable points emerge at $x = \pm\sqrt{r}$

3. Hopf Bifurcation

Equation (2D system):

$$\begin{cases} \frac{dx}{dt} = rx - y - x(x^2 + y^2) \\ \frac{dy}{dt} = x + ry - y(x^2 + y^2) \end{cases}$$

Behavior:

- For $r < 0$: stable spiral at origin
- For $r > 0$: unstable spiral at origin and a stable limit cycle emerges

4. Transcritical Bifurcation

Equation:

$$\frac{dx}{dt} = rx - x^2$$

Behavior:

- Fixed points at $x = 0$ and $x = r$
- Their stability switches at $r = 0$:
  - For $r < 0$: $x = 0$ is stable, $x = r$ unstable
  - For $r > 0$: $x = 0$ is unstable, $x = r$ stabl